EXPERIMENTS ON THE RATE OF HEAT TRANSFER FROM A HOT GAS TO A COOLER METALLIC SURFACE

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FIG. 1

INTRODUCTION

WHILE there has been within recent years a number of experiments dealing with the subject of "Heat Transmission," it is apparent from a study of the literature on the subject that not only is there a marked uncertainty as to the actual value of the numerical coefficients involved, but also a considerable lack of information with reference to the theory. The 35th Edition of "Steam"* gives a brief account of what was known at the end of the year 1913 of the laws governing the transfer of heat from a hot gas to a cooler metallic surface, and since that time there have been practically no additional contributions to the literature on the subject.

The experiments of H. P. Jordan, an account of which is given in the Proceedings of the Institute of Mechanical Engineers, 1909, appear to be the most reliable and are, in fact, the only experiments that seem to offer anything from the viewpoint of practical design. The methods employed by Jordan in working up his results corresponded with the methods used by The Babcock & Wilcox Co. some years ago in the determination of average heat transfer rates in boilers from evaporative tests. Jordan's results, however, and those of The Babcock & Wilcox Co. differed very materially in the numerical value of the constants determined. Because of this wide discrepancy in results, and with a view to arriving at some definite conclusions on the laws governing heat transfer as applied to actual practice, as distinguished from the purely theoretical aspect of the subject, the experiments described in this paper were undertaken.

Before criticising Jordan's results it would be well to point out a few deductions from the elementary theory covering the transfer of heat. As ordinarily used, the heat transfer rate is equal to

$$\frac{\text{Quantity of heat}}{\text{Temperature difference x Area x Time}}$$

A quantity of heat dimensionally is equal to the product of a mass times a temperature difference; hence, the ratio

$$\frac{\text{Quantity of heat}}{\text{Temperature difference}}$$

*"Steam, Its Generation and Use," 35th Edition, 1913. The Babcock & Wilcox Co.

has the dimension M, and the area and time, of course, have the dimensions L^2 and T. Therefore the dimension of the transfer rate coefficient is

$$ML^{-2}T^{-1}$$

The dimension of the coefficient of conductivity, as ordinarily used, is obviously

$$ML^{-1}T^{-1}$$

Hence, considered dimensionally, the transfer rate could be equal to a coefficient of conductivity divided by a length, which in the case of a cylindrical tube must be the diameter of the tube. It is also evident that from the dimensions the transfer rate can be equal to a velocity times a density, and again, considering a flow through a cylindrical tube, the velocity and density can be taken as the mean density of the gas, which gives Osborne Reynolds' law.

It has been recently demonstrated by the experiments of Stanton and Pannell* that the motion of a viscous fluid of constant viscosity depends only on the coefficient of viscosity and the density. When the fluid is a gas varying in temperature and density, other coefficients are introduced, but it seems impossible to combine any of these coefficients in a rational way to give a term that would not depend upon the diameter of the tube, other than the one mentioned, namely, velocity times density. If, therefore, the transfer rate R be written

$$R = a + b\frac{W}{A} \qquad (1)$$

where W = the weight of gas passing through absorbing surface per hour,

A = area of the section of the channel through which the gas passes, in square feet,

it is natural to assume that the term a would depend upon the coefficient of conductivity of the fluid at the temperature of the wall of the tube divided by the diameter of the tube. The term $\frac{W}{A}$ has the same dimensions as velocity multiplied by density, so that the coefficient b is of zero dimension and can, therefore, be a function of the ratios of any two quantities of the same dimension that could be involved in the equations of motion of the

* Transactions of the Royal Society, Vol. 214, 1914.

fluid or the transfer of heat. The value of b could thus vary with the diameter of the tube or with the temperature difference, and considering the approximate nature of the formula, would probably vary if extended over a wide range with the mean velocity of flow.

The formula determined by The Babcock & Wilcox Co. for the average transfer rate with Babcock & Wilcox boilers gives a in (1) the value 2.0 and b the value 0.0014, while Jordan's formula gives a the value 5.4 and

$$b = .000506 - .00045\, r + .0000083(T + t)$$

where r = the area of channel divided by the circumference in inches,

T = the mean temperature of the gas, degrees Fahrenheit,

t = temperature of the absorbing surface.

Jordan's results apparently indicate that the value of the constant a is independent of the shape and size of the channel for the passage of the gases, and is independent of the temperature difference. It is difficult to account for such a fact, particularly as the value of the constant a, as determined by The Babcock & Wilcox Co., was less than one-half as large as Jordan's value. At very high gas velocities, the second term in the transfer rate formula (1) is the more important, but with the velocities found in ordinary boiler practice the terms are of about equal importance. From the standpoint of design, therefore, it is necessary to know how each term is affected by the change in the temperature of the gas, in that of the absorbing surface, and in the dimensions of the gas passage channel.

As stated, the object of the present experiments was to explain the discrepancy between Jordan's conclusions and those arrived at by The Babcock & Wilcox Co. in determining the average heat transfer rate with Babcock & Wilcox boilers, and since the results of these experiments do not check with Jordan's, it will be well to examine the difference in the plan of the two sets of measurements in order to arrive at a conclusion as to which is the more accurate.

The article on "Heat Transfer" in "Steam," to which reference has been made, points out that the temperature of the gas used in any heat transfer formula requires definition.

The temperature of a gas flowing through a channel will vary across the section of the channel, being a maximum at the center. Professor Gröber, in an article published in the "Zeitschrift des Vereines Deutscher Ingenieur," in March, 1912, gives the results of some measurements of temperature difference across a cylindrical tube, which would indicate that the mean of the actual readings taken at about equal intervals is approximately 75 per cent of the temperature difference at the center of the tube. The mean temperature taken in this way, however, is not the mean temperature that is used in any formula connected with heat transfer. In such formulae, what is required is the mean temperature of the gas flowing past a given cross section. To obtain such a mean from temperature measurements such as Gröber's, each measurement would have to be multiplied by the product of the velocity and density of the gas at the particular point at which it was taken and by the distance from the center, and the total sum of these products divided by the sum of the multipliers of the several measurements. It is obvious, therefore, that it would be impossible to actually measure the mean temperature of a gas at any cross section of a tube with any degree of accuracy. In fact, considering the effect of the radiation absorbed by the cooler surface of the tube, any attempt to measure the temperature of the gas in the tube itself will be out of the question if accuracy is desired, and such temperature measurements might even be so far out as to lead to erroneous general conclusions.

The mean temperature of a gas inside of any vessel, the walls of which are at the temperature of the gas, would be shown by a thermometer placed in the vessel and surrounded by the gas. Mean temperatures could be measured in a more or less closed vessel with insulated walls before entering and after leaving calorimeter tubes, such as were used by Jordan, and, for the purpose of determining specific heat, such methods would be correct. When, however, the quantity sought is a heat transfer rate, there are other conditions that would condemn such a method. Since the transfer rate is known to depend upon the motion of the cooling gas, it is obvious, to give definiteness, that a uniform motion of some sort with a gradual gradation in temperature between the hottest portion of the gas and the tube be specified. Measuring the transfer rate in a tube takes this fact

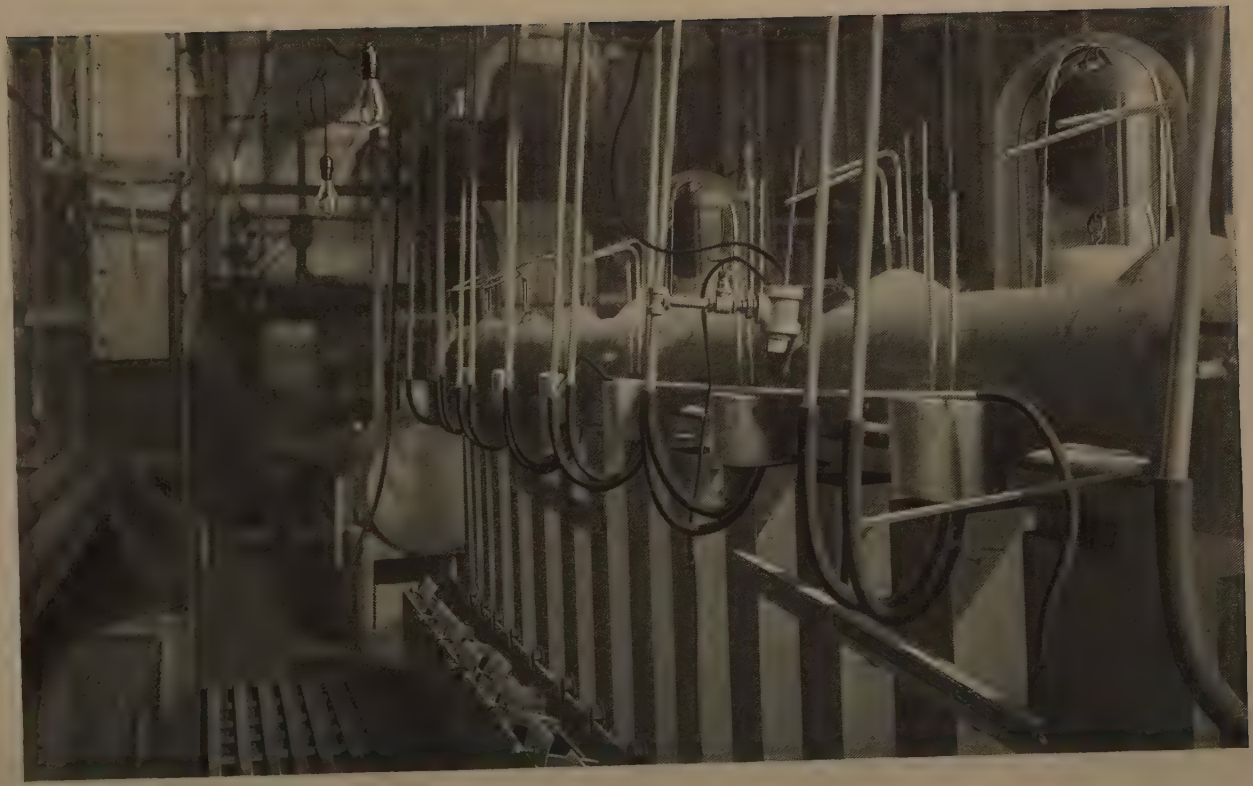


FIG. 2

into consideration, provided the motion is modified simply by the cooling of the gas. There is, however, no *prima facie* evidence that the transfer rate should not depend upon the past history of the gas motion and therefore be connected with the length of the tube. It must, therefore, be assumed, as established by experiment, that when the motion and temperature gradients are such as would be established after the end or entrance disturbances have died out, the transfer rate will depend only upon the mean velocity and temperature of the gas and not upon its past history. Obviously, it is necessary to have a motion in an experimental apparatus through a short length of channel before the mean temperature used in the transfer rate formula can be determined. This elementary consideration will show that it would be impossible to measure the mean temperature in an insulated chamber before the gas entered the tube since, granting that the motion was at once established, the mean temperature measured in this chamber could only be the maximum temperature of the gas at the center of the tube, the actual mean temperature in the tube being much lower, and it would follow that a hot gas suddenly entering a channel between cooling surfaces would at first give out heat at a much more rapid rate than after a steady motion is established with a less steep temperature gradient.

From the foregoing it is very evident that Jordan's methods of measuring temperatures are open to criticism and, in fact, any other method of measuring temperatures of gases surrounded by cooling surfaces that depend upon the readings of a thermometer or thermo-couple are equally open to criticism unless the temperature so measured is determined by quickly conducting the gases away from the cooling surface, mixing it thoroughly, and then recording the mean temperature. While it is impossible to obtain an accurate mean temperature measurement in a heat absorbing tube itself or from the gas before it enters such a tube, it is possible to get a true mean temperature with the thermometer in the gases leaving the heat absorbing tube, provided that such a measurement is corrected for radiation. It follows that when the object is the determination of the heat transfer rate, the only method of determining mean temperatures during the process of cooling the gas is to determine the heat given up by the gas and in turn compute from this the drop in temperature, which is to

be added to a known lower temperature. In the experiments under consideration, the total heat given up by the gas was divided into twenty parts and was separately measured, and the mean temperature of the gas through different sections of the experimental heat absorption tube determined in this way. A criticism of this method might be that a knowledge of the specific heat of the gas is involved. While such a criticism would have been of considerable importance some years ago, it is not the case since the experimental determination of the specific heat of nitrogen, carbon dioxide and water vapor by Holborn and Henning. (See Appendix C.)

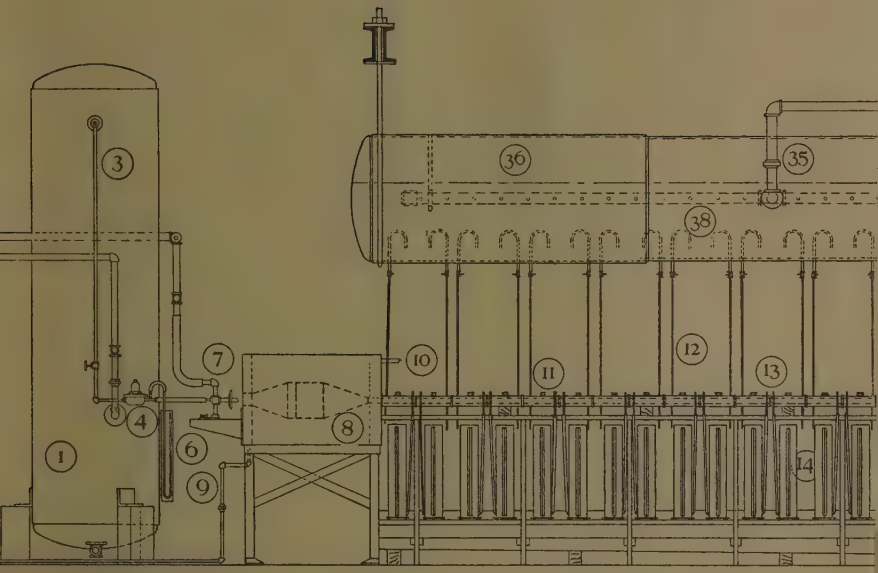
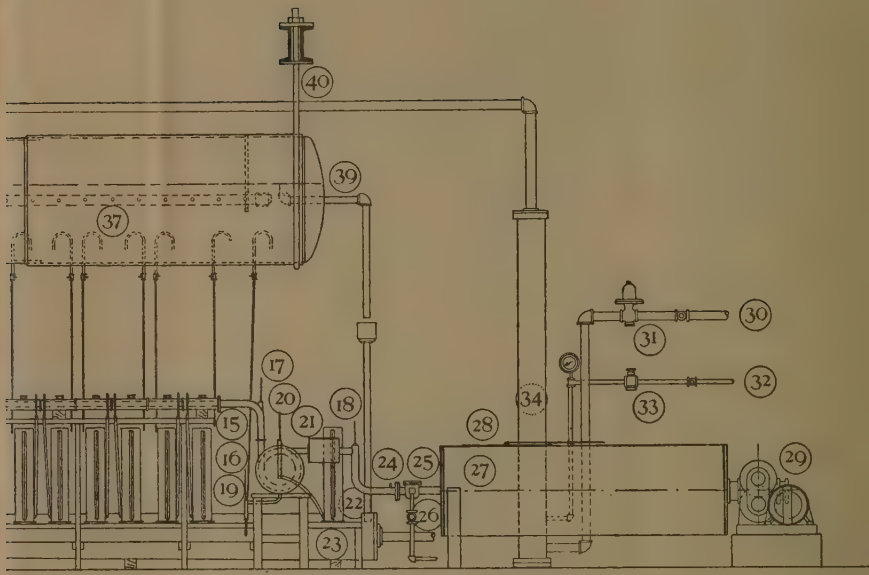


FIG. 3. GENERAL ARRANGEMENT OF EXPE

1 COMPRESSED AIR RESERVOIR
2 ENTERING AIR SUPPLY
3 AIR LEAVING RESERVOIR
4 FOSTER PRESSURE REGULATOR
5 ILLUMINATING GAS SUPPLY
6 MERCURY U TUBE
7 GAS BURNER
8 FURNACE
9 COOLING WATER INLET
10 COOLING WATER OUTLET
11 HEAT TRANSFER TUBE
12 WATER INLETS TO JACKETS
13 WATER OUTLETS TO TANKS
14 WATER MEASURING TANKS
15 CURVED PIPE TO GAS COOLER
16 GAS COOLER
17 GAS IN THERMOMETER
18 GAS OUT THERMOMETER
19 INLET WATER THERMOMETER
20 OUTLET WATER THERMOMETER



RIMENTAL HEAT TRANSFER APPARATUS

21 EXPANSION CHAMBER
22 MEASURING TANK FOR COOLER
23 WATER TROUGH
24 CONNECTION FOR GAS SAMPLE
25 DEW POINT APPARATUS
26 WATER SUPPLY FOR DEW BOX
27 EQUALIZING CHAMBER
28 SLOT FOR CONTROLLING DRAFT
29 GAS EXHAUSTER
30 COLD WATER SUPPLY
31 FOSTER PRESSURE REGULATOR
32 ENTERING STEAM SUPPLY
33 LESLIE PRESSURE REGULATOR
34 WATER HEATER
35 HOT WATER ENTERING DRUM
36 OVERHEAD SUPPLY DRUM
37 PERFORATED DISTRIBUTING PIPE
38 PIPE LOOPS
39 SPILLWAY
40 SUSPENSION STRAPS

DESCRIPTION OF THE EXPERIMENTAL APPARATUS

A GENERAL view of the apparatus with which these experiments were conducted is shown in Figs. 1 and 2 and diagrammatically in Fig. 3. It consists of a 20-foot copper pipe of 2-inch internal diameter surrounded by twenty individual water jackets. An illuminating gas furnace supplied the heated gases, these being drawn through the pipe by suction. After passing through the experimental transfer tube, the gases passed through a curved pipe to a helical pipe cooler surrounded by water. From the cooler, the gases passed through a dew point box into an equalizing chamber, from which they were discharged into the atmosphere through a gas exhauster. The circulating water for the jackets and cooler was supplied from an overhead tank.

A brief statement of the methods followed in making calculations, as a means to pointing out the functions of the different parts of the apparatus, will be of assistance in following the detailed description. The methods of such calculations are described fully hereafter.

The determination of the dew point of the gases after leaving the cooler determines the density of the water vapor in the gases. With the dew point known and an analysis of the gases passing through the dew point box, the proportionate parts by weight of the various constituents of the gas may be determined. This in turn, together with the gas temperature entering and leaving the cooler, enables the mean specific heat of the gases through the cooler to be calculated. With the specific heat, the temperature of the gases entering and leaving the cooler, the temperature of water entering and leaving the cooler, and the weight of water passing through the cooler known, the weight of the gases may be calculated.

As stated in the Introduction, in the determination of the heat transfer rate, the total heat given up by the gases is divided into twenty parts and measured separately.

The temperature of the gases entering the cooler (or leaving jacket No. 20) is known. The weight of water passing through this jacket and the temperature of water entering and leaving

this jacket are known. The instantaneous specific heat of the gases at the temperature taken leaving jacket No. 20 is assumed to be the mean specific heat of the gases in their passage through this section of the transfer tube. From these factors the drop in temperature of the gases through this section and the entering temperature may be determined.

The temperature of the gases entering jacket No. 20 is the temperature of the gases leaving jacket No. 19. The instantaneous specific heat of the gases at this point is assumed to be the mean

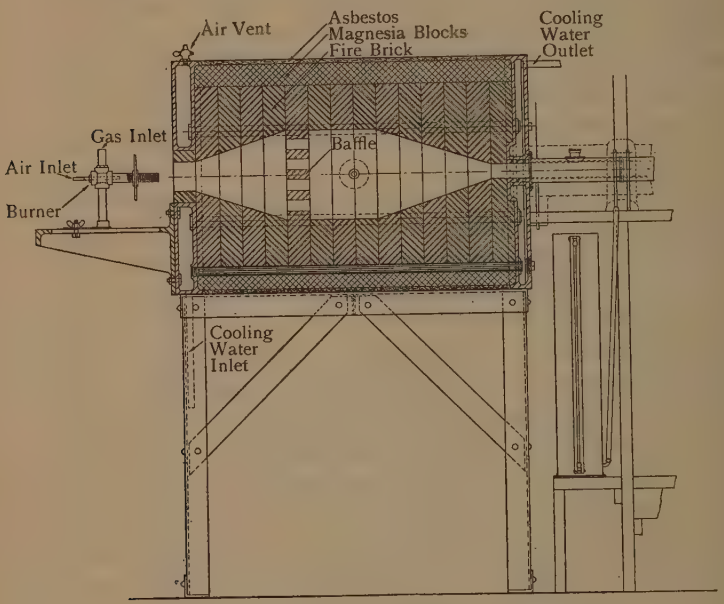


FIG. 4. CONSTRUCTION OF FURNACE

specific heat of the gases in their passage through jacket No. 19. On this assumption the drop in temperature through jacket No. 19 and the entering temperature of the gases into section No. 19 may be determined. This method was followed back to jacket No. 1 next to the furnace.

THE FURNACE—The form of the furnace is shown in Fig. 4. This was built of firebrick throughout, set in a water-cooled metal

box, the whole being thoroughly insulated. The design was such as to insure a thorough mixture of gas and air.

The gas burner (see Fig. 4), the position of which was adjustable, consisted of a one-inch iron cross. Illuminating gas under a steady pressure of approximately 3 inches of water was supplied through the top. Compressed air from the reservoir shown in Fig. 3 entered through a ¼-inch pipe at the left of the cross. This pipe extended into a one-inch delivery pipe so that gas and air were mixed before leaving the burner. A plate, 6 inches in diameter, was threaded on the delivery pipe giving a means of adjusting the admission of atmospheric air to the furnace.

The compressed air pressure was kept constant by a Foster pressure regulator.

By regulating the gas and compressed air supplies and adjusting the plate controlling the atmospheric air supply, variation in gas temperature and in the gas analysis could be secured. Records of gas used, air pressure and atmospheric air openings were kept in order that any set of conditions could be duplicated.

Heat Transfer Tube and Jackets.—Fig. 5. The gas passage was formed by a seamless hard drawn copper tube 20 feet long, 2 inches internal diameter and 0.134 inch thick. This tube was surrounded by twenty individual water jackets, each approximately one foot long. These jackets were made of copper flanges ⅛ inch thick and seamless copper pipe of 3 inches internal diameter and 0.083 inch thick. Each flanged jacket was soldered to the transfer tube, as shown in Fig. 5. Individual jackets were separated from each other by asbestos gaskets.

Semi-circular baffles of copper were installed in the annular spaces between the transfer tube and jackets to insure a definite flow of water through each jacket. This construction is clearly shown in Fig. 5. All jackets were tested hydrostatically under 4 pounds pressure to obviate the possibility of any leakage.

The water inlet and outlet pipes to jackets were made of copper tubing ½ inch internal diameter. The inlet pipe of jacket No. 1 was nearest the furnace and at the bottom of the jacket. The outlet pipe was at the top and 10½ inches away. The outlet for jacket No. 2 was adjacent to the outlet for jacket No. 1 and the inlet for this jacket was at the bottom 10½ inches away. This inlet and outlet connection construction holds good throughout

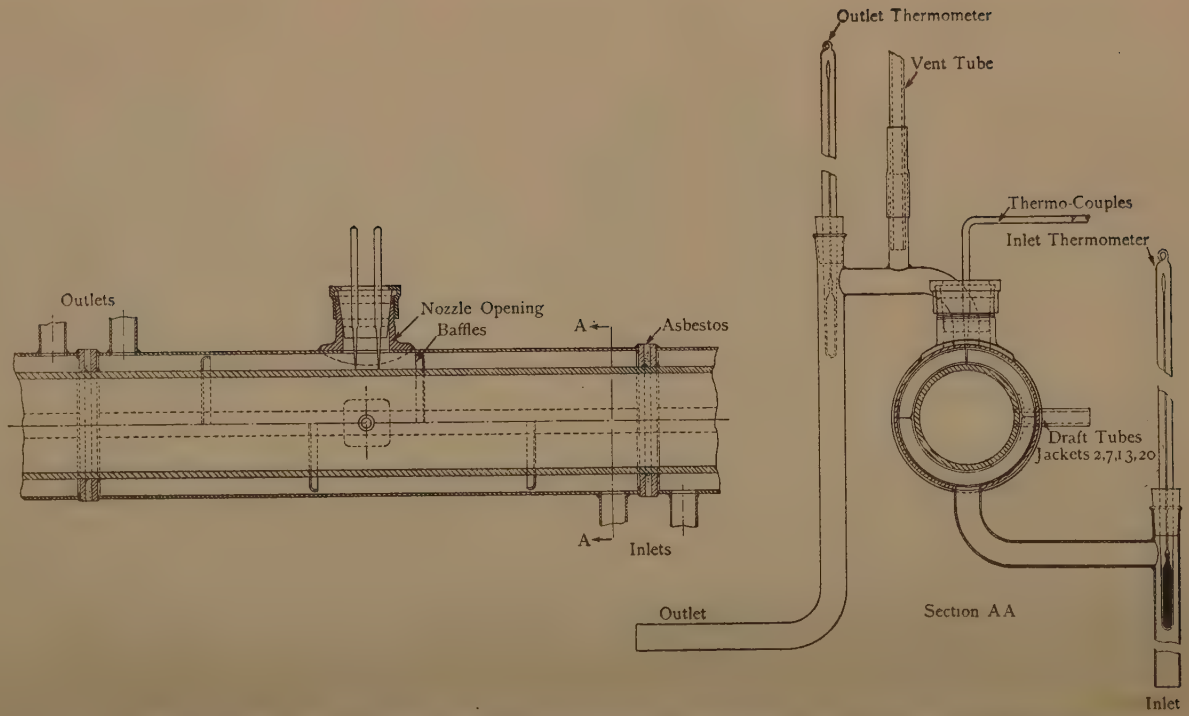


FIG. 5 CONSTRUCTION OF JACKETS AND ARRANGEMENT OF INLET AND OUTLET THERMOMETERS

the tube so that inlet and outlet connections are always in pairs. As will be shown, the temperature of water entering all jackets was practically the same, so that there could be no flow of heat from one jacket to adjacent jackets.

The outlet connection to each jacket was provided with a glass vent tube in order that the water level might be maintained above the jacket, and to assure that the jackets were full of water at all times.

The methods of holding thermometers in the inlet and outlet connections is clear from the illustration. On top of each jacket, midway between flanges, one-inch brass nozzles with screwed caps were soldered. These openings were employed as receptacles for the thermo-electric couples used in determining the metal temperatures. These will be described in Section VI, "Measurement of Metal Temperatures."

The jackets were covered with 2½-inch magnesia to minimize the radiation loss. The whole was mounted on a wooden stand in such a manner that the opening from the furnace and the tube were continuous.

Water Supply—The water for absorbing heat from the jackets and cooler was supplied from a drum 42 inches in diameter, 23 feet long, suspended approximately 5 feet 9 inches above the center line of the transfer tube. A mixing chamber 8 inches in diameter by 8 feet high in which steam and water were mixed to give the desired temperature was placed on the supply line to the tank. The water pressure was kept constant by a Foster pressure regulator and steam was delivered at any desired pressure by a Leslie pressure regulator.

The water entered the overhead tank through the side of the shell and was distributed by means of a T and pipe connections extending over the whole length supplied with small feed holes. In this way the temperature was a constant throughout the length of the tank and the maximum variation in temperature of water entering all jackets was not in excess of one degree. A spillway 5 inches above the center line of the tank kept the water level constant at all times. Water was taken from the supply tank to individual jackets and the cooler through wrought-iron pipes provided with loops inside the tank and drawing water from a point 4 inches above the bottom of the tank, in this way

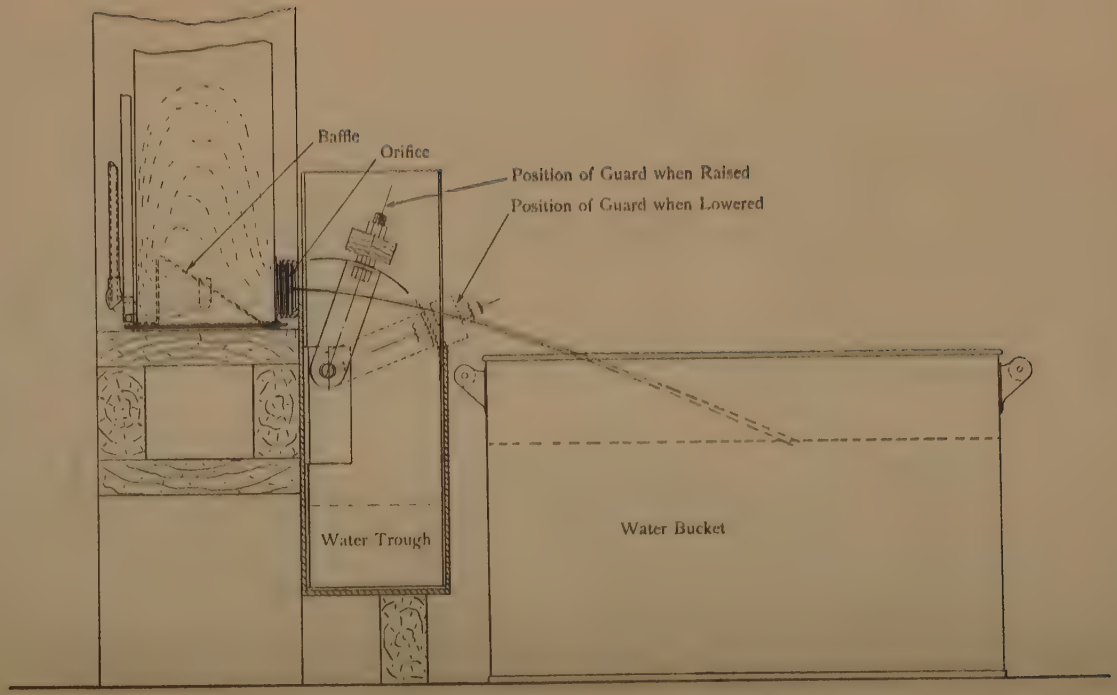


FIG. 6. CONSTRUCTION OF APPARATUS TO CONTROL FLOW OF WATER

preventing the accumulaton of scale or dirt in the jackets. The outlets from the tank were provided with ¾-inch stop cocks and connections were made from the outlet pipes to the jacket inlet pipes by rubber hose.

Water Weighing Apparatus—Fig. 3. Directly beneath the heat transfer tube and jackets, on the same stand, twenty galvanized iron tanks, 6 inches square and 2 feet 8 inches high, were placed. Each tank was provided with a gauge glass and a scale graduated to decimals of one foot for reading water levels. The water from the jacket outlets passed through rubber hose to these tanks, the inlets being near the bottom. The outlet from each tank was near the bottom on the side opposite the gauge glass. These outlet tanks were fitted with interchangeable monel metal orifices. Baffle plates at the bottom of the tanks prevented any disturbances of water in the neighborhood of the orifices. As stated, in order to keep the temperature of the water from adjacent jackets approximately the same, the rate of flow through different jackets was varied. This was accomplished by the use of pinch cocks on the connecting hose to regulate the flow of water. The size of orifice was then used that would give a perfect jet from the tank.

To the supports below each tank, there was hinged a guard. With the guard lowered, the stream of water from an orifice was deflected into a sheet-iron trough extending the whole length of the apparatus and connected to the return system. With the guard raised, the stream from the orifice flowed into a bucket in which the water flowing for any given period could be weighed. The twenty guards were connected in such a manner that all could be raised or lowered as desired. This construction can be readily seen from Fig. 6.

The Gas Cooler—The gases after leaving the heat transfer tube passed through a curved pipe of 2 inches internal diameter connected to the cooler. The temperature of the gas leaving the last jacket and entering the cooler was measured in this curved pipe. The cooler consisted of a helical coil of 2 inches internal diameter pipe approximately 26 feet long and placed inside of a water drum 16 inches in diameter and 30 inches long. The cooling water entered the cooler at the bottom and left at the top of the water drum. A vent tube in the outlet pipe

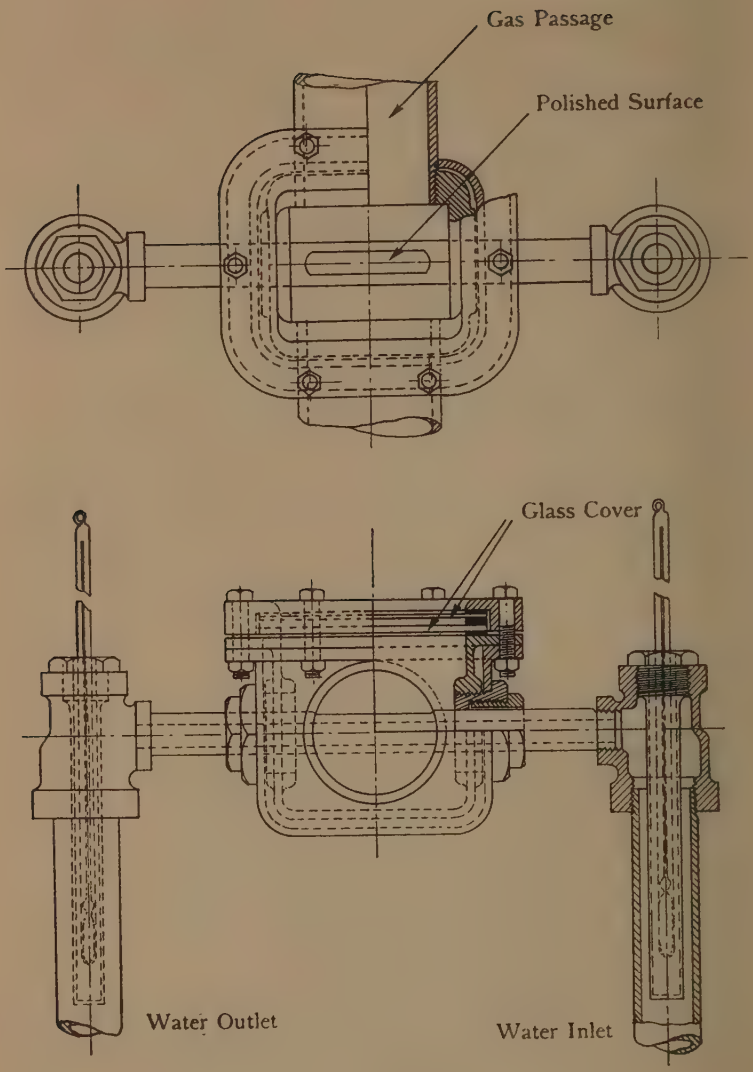


FIG. 7 DEW POINT APPARATUS

assured the cooler being filled with water at all times. The water leaving the cooler passed through a tank similar to that supplied for individual jackets and the same arrangement of pivoted guard was provided for deflecting the stream of water from the tank. The temperatures of water entering and leaving the cooler drum were taken by thermometers held in the same manner as those in the inlet and outlet jacket connections. The whole cooler was covered with 3-inch magnesia to reduce radiation losses.

Dew Point Apparatus—After leaving the cooler, the gases passed through the dew point box shown in Fig. 7. This box was made of brass and was fitted with a gas-tight glass cover through which a highly polished nickel pipe crossing the box at right angles to the direction of the gas flow could be observed. The upper side of this pipe was flattened and served as a mirror. Cold and hot water supplies were connected to one end of this pipe so that the water passing through it could be made of any desired temperature. When the temperature of the water was lowered to the dew point of the gas, its water vapor content would begin to condense, an action that could be noted on the polished flat surface of the nickel tube with the aid of a magnifying glass.

The temperatures of the water entering and leaving the pipe were taken by thermometers immersed in mercury wells. Just before the dew point box a small opening was provided for drawing the gas samples for analysis.

Equalizing Chamber—After leaving the dew point box the gases passed to an equalizing chamber 28 inches in diameter and approximately 7 feet 6 inches long. The end of this chamber was connected to a Root exhauster geared to a 2½ horse-power motor. This equalizing chamber was used to obviate any pulsating action of the gas in the apparatus due to the action of the blower.

A slot 12 inches long by one inch wide on the top of the equalizing chamber was fitted with an adjustable slide. By regulating the position of this slide, the amount of air that could be drawn into the equalizing chamber was varied, and this, together with the different motor speeds, made possible any desired draft suction through the heat transfer tube.

The gases leaving the exhauster passed direct to the atmosphere.

The barometer used was of the standard United States Weather Bureau type.

A sling phsychrometer was used to determine the relative humidity of the air.

The platform scale used to weigh the water discharged into the buckets was provided with agate bearings and was sensible to one-quarter ounce, which is approximately one-hundredth of one per cent of the lowest amount of water weighed.

INSTRUMENTS—A description of the thermo-couples used to measure metal temperatures along the transfer tube is given in Section VI.

The thermometers used to measure the inlet and outlet water temperatures in the jackets and the cooler were especially made for these experiments by the Hohman & Maurer Manufacturing Company. These thermometers were approximately 14 inches long and $\frac{5}{16}$ inch in diameter and were graduated to 0.1 degree Fahrenheit, the scale being such that 0.01 degree Fahrenheit could be estimated. Thirty thermometers were scaled from 130 to 170 degrees Fahrenheit, ten from 140 to 180 degrees Fahrenheit, and ten from 150 to 190 degrees Fahrenheit. The thermometers were all calibrated and found to be correct within the limits of error of the experiments. The thermometers were observed to the nearest 0.1 degree Fahrenheit.

The thermometers used to measure the inlet and outlet gas temperatures to the cooler were graduated to one degree Fahrenheit and temperatures were recorded to the nearest one-half of one degree. Standard thermometers, made by H. J. Green, were used for taking room temperatures and the dew point temperature of the gas.

PRELIMINARY EXPERIMENTS

PRELIMINARY experiments, covering a period of over three months, were conducted with a view to obtaining a full knowledge of the apparatus and an assurance that the various parts would properly fulfill their intended functions. These experiments brought to light several factors that made necessary some slight modifications in the construction and the methods of operation originally proposed.

It was the intention in the original design of the apparatus to use water at room temperatures for circulation through the jackets and the cooler in order to assure an approximately constant temperature of water supply and to obviate the necessity for any radiation correction. It was found when water at room temperatures was used that the gas temperature in the last few jackets was reduced below the dew point of the gas. The resulting formation of moisture made it impossible to determine accurately the heat given up by the gases to the cooling water through this section of the transfer tube. It was practically impossible with the apparatus as designed to weigh this moisture or to state accurately just where the formation of moisture started. Further, the moisture in the gases carried at the velocity of some 40 feet per second, striking the bulb of the thermometer measuring the gas temperature entering the cooler, caused it to crack. To overcome these difficulties the water supply was heated to approximately 145 degrees (or about 15 degrees above the highest dew point recorded) by means of the addition of the steam and water mixing chamber in the supply line. To assure a constant temperature of water throughout the overhead drum, changes were made in the details to bring it to the design already described.

The use of a heated water supply to the jackets and cooler made necessary the application of a radiation correction. The methods of determining radiation losses from different jackets is described under "Radiation Tests," page 30.

It has been indicated in the Introduction that while it is impossible, for purposes of heat transfer determination, to obtain a true mean temperature measurement of a gas in the transfer tube or before entering the transfer tube, it is possible to get a proper

mean temperature with the thermometer in the gas leaving a heat absorbing tube.

Only two gas temperatures were recorded in the present experiments and these were taken after the gases had passed from the absorbing tube. These temperatures were: first, that of the gases leaving the last jacket or entering the cooler; second, that of the gases leaving the cooler.

The thermometer recording the inlet gas temperature to the cooler was located (Fig. 3) in the curved pipe connecting the transfer tube with the cooler, the bulb being in direct contact with the gases. No change of gas flow occurred in the vicinity of this thermometer, for the pipe conducting the hot gases had a 2-inch internal diameter for a length of some 48 feet and the thermometer bulb was 21 feet from the furnace end. Further, the thermometer bulb was one foot from the nearest water cooling surface, eliminating possible error from such a source. Inasmuch as this portion of the apparatus was covered with 3-inch insulating material, it was not necessary to apply any radiation corrections to the inlet gas temperatures recorded. The highest temperature measured throughout the experiments was 570 degrees, while the average was below 500 degrees.

The proper immersion of the thermometer to give the mean temperature of the inlet gas was determined by a set of tests run for this specific purpose. Under constant conditions of gas flow the bulb of the thermometer was placed in different positions in the curved pipe by varying the amount of immersion. The average of the readings secured in a great number of positions was taken as the mean temperature of the gas across the section of the pipe. The thermometer was then immersed the proper amount to give this average reading and the degree of immersion recorded. This immersion was then used in all tests. The variation of the temperatures for different positions of the thermometer bulb was small and the effect on the results of the maximum variation noted would be well within the error of the experiments as a whole.

The exit temperature of the gases from the cooler was measured by a thermometer located in the expansion chamber between the cooler and the dew point box. The temperature of the gases at this point was approximately 150 degrees and

no variation of temperature could be found by varying the location of the thermometer bulb.

It was the original intention to determine the weights of water flowing through the jackets and cooler by means of calibrated orifices from the water tanks and the heads in these tanks. It was found that sediment collected slowly on the edges of the orifices and this fact, together with the slight variation in water temperatures from test to test, made it obvious that water weights calculated by such a method would not be of sufficient accuracy. For this reason it was decided to actually weigh the water, as described, for a definite period during the test. By maintaining the water levels in the various tanks constant, the rates of water flow for any day could be accurately determined from the weights so secured, regardless of any reduction in the size of the orifice due to accumulation of sediment on the orifice.

As has been stated, the temperature of the water entering the various jackets was practically uniform. While the exit temperatures of water from any two adjacent jackets were so nearly alike as to obviate the possibility of any flow of heat from jacket to jacket, still, because of the variation in gas temperature in different jackets, these exit temperatures could not be made exactly uniform throughout the whole apparatus. To make these temperatures as nearly uniform as possible, the rates of water flow through successive jackets were decreased and this necessitated experiments with different sizes of orifices to determine which would give perfect jets from the individual tanks to the weighing buckets for these different rates of flow. These orifices were graduated from $\frac{5}{32}$ inch to $\frac{3}{8}$ inch, the sizes for the different tanks being given on the data sheet.

The polished pipe at first used in the dew point box was of copper and nickel composition. The flattened surface of this pipe showed a tendency to tarnish, possibly from a corrosive action of the flue gases on the copper component, and after a short time rendered difficult the accurate determination of the dew point. This trouble was completely eliminated by the substitution of a pure nickel pipe for the composition pipe.

In the original design of the apparatus the dew point box was connected to the exhauster by a conical pipe of the same

length as the equalizing chamber already described but with a diameter of 2 inches at the dew point box end and 6 inches at the exhauster end. Because of the relatively small volume of this conical connection, it was found that the pulsations of the fan caused an appreciable fluctuation of the gases throughout the apparatus. This pulsating effect was overcome and a steady suction secured by replacing the conical pipe by the 28-inch diameter equalizing chamber.

METHOD OF CONDUCTING EXPERIMENTS AND RECORDING DATA

THE data throughout the experiments was recorded by five observers, to each of whom were assigned definite readings.

The apparatus was in operation throughout the whole of each day. The morning period was devoted to such adjustments of the apparatus as would bring about steady conditions, such conditions, insofar as the rate of gas flow was concerned, being predetermined. The data from which computations were made was taken in the afternoon after the steady conditions had been maintained over a period of three or four hours.

In the morning the water supply was heated to 140-145 degrees and allowed to flow through the jackets and the cooler. The nickel pipe in the dew point box was polished and the draft gauges adjusted to zero. The gas burner was then started and the furnace gradually heated. The rate of gas flow was dependent upon the illuminating gas consumption and the external air admitted. The gas consumption was recorded every half hour and the compressed air, which was mixed with the gas, was supplied at a constant pressure. The percentage of CO_2, as determined by analysis, indicated the amount of external air supplied for combustion and the desired amount was secured by adjusting the suction from the exhauster by means of the slot on the equalizing chamber, thus regulating the amount of air drawn in around the burner. That the desired steady conditions were secured was checked by frequent gas analysis and observations of the draft gauge attached to the transfer tube through the twentieth jacket.

After the furnace conditions and rate of gas flow were constant, the rates of water flow to the different jackets were adjusted to give the same exit water temperatures from two adjacent jackets. The water levels in the glass vent tubes were kept in sight at all times. As soon as the desired rates of water flow were secured, the heads of water above the orifices in the tanks became steady. With all conditions thus made constant, the apparatus was ready for a test run. The method of taking data was as follows:

Before the water was allowed to flow into the weighing buckets, a gas analysis was made and dew point of the gases determined. One observer watched the polished surface of the nickel pipe. The temperature of the water flowing through this pipe was decreased very slowly and at the instant moisture began to form, the temperatures in and out of the pipe were recorded. The barometer was then adjusted and observed, the room temperature taken from thermometers at either end of the apparatus and readings of the sling phsychrometer taken near the furnace, near the cooler, and at the center of the apparatus. The heads of water above the orifices in the tanks were recorded by two observers immediately before the water flow to the weighing buckets was started. One observer started at tank No. 1 and the other at tank No. 11, each observing all twenty tanks, in this way securing two sets of readings. The guards were then raised and the water allowed to flow into the weighing buckets for six minutes.

During this period inlet and outlet temperatures to jackets were recorded as follows: Two observers, stationed on the opposite sides of jacket No. 1 read the inlet and outlet temperatures at the same time. Two other observers were similarly stationed at jacket No. 11. The first two read the twenty jackets in consecutive order and came back to the first jacket in slightly less than three minutes, while the second two passed from jacket No. 11 to jacket No. 20 consecutively, then to jacket No. 1 and up to No. 11 in the same time. This operation was repeated before the water flow to the weighing buckets was stopped, and in this way each inlet and each outlet temperature was observed and recorded four times in succession by two different observers. The fifth observer during these six minutes recorded the inlet and outlet water and gas temperatures to and from the cooler and started and stopped the water flow from tank No. 21 to the weighing bucket at the same time as the flows from the other jackets.

The heads in the tanks, the dew point, barometer, and room temperatures were again observed and the flue gas analyzed. The water in the buckets was then weighed and the test was completed. All data was recorded in numbered log books and later transferred to log sheets, as shown in Table 1.*

Ordinarily three tests were run each afternoon under constant gas conditions. During the whole afternoon a sample of

* Page 50.

illuminating gas was taken for analysis. After these tests, the furnace was shut down and allowed to cool before stopping the flow of water through the jackets and cooler.

In all, 106 tests were made in this manner. The range of gas consumption was from 2.5 to 8.6 cubic feet per minute and the percentage of carbon dioxide varied from 9.6 per cent to 11.5 per cent. These figures correspond to a rate of gas flow from 4100 to 14,300 pounds per square foot of cross sectional area of the transfer tube per hour.

DETERMINATION OF RADIATION LOSSES IN JACKETS AND COOLER

WITH the transfer tube through which the gases were drawn sealed, provided no heat is given up to the confined air, any decrease in the temperature of the water flowing through the jackets and the cooler must represent a loss due to radiation. The difference in the temperatures between the inlet and outlet water to the jacket multiplied by the weight of water flowing through that jacket per hour will give the radiation loss in B. t. u.'s per hour and this method was utilized in determining the correction to be applied for such radiation loss.

The tube was sealed at both ends and water was circulated through the jackets and the cooler under conditions as nearly as possible like those existing in the test runs, that is, conditions of entering temperatures and rates of water flow. Before recording any data, the rate of water flow was maintained for a period sufficient to allow the air confined in the tube to reach its maximum temperature. The inlet and outlet thermometers were read at the same time to 0.01 degree Fahrenheit. The losses in jackets Nos. 13 to 19, inclusive, were used to determine the average loss because the rate of water flow through these jackets was sufficiently low to give a good temperature difference. The weight of water was obtained in the usual manner.

Two sets of radiation tests were run on four separate days, giving eight tests of seven jackets, or fifty-six tests in all. The final temperature differences recorded for each individual test were the average of from six to ten temperature readings.

The average loss per jacket from these fifty-six tests was 31.4 B. t. u.'s per hour and in making computations, 31 B. t. u.'s were added to the heat absorbed in each jacket as a radiation correction. This correction is at the rate of, approximately, 0.19 B. t. u. per square foot of surface per degree difference in temperature between the jacket and the external air, a figure which is in line with other available data on radiation losses. Such a correction represents, in terms of the total heat absorbed by jacket No. 2, from 0.6 per cent for the lowest rate of gas flow to 0.01 per cent for the highest rate, and for jacket No. 19

from 6.3 per cent for the lowest rate of gas flow to 0.1 per cent for the highest rate of gas flow.

The radiation loss from the cooler was determined in the same way. The average loss was 266.9 B. t. u.'s per hour and in making computations 267 B. t. u.'s were added to the heat absorbed per hour by the cooler as a radiation correction. This correction represents, in terms of the total heat absorbed by the cooler, from 7.0 per cent for the lowest rate of gas flow to 0.9 per cent for the highest rate of gas flow.

MEASUREMENT OF METAL TEMPERATURES

AS has been indicated, the method of the determination of heat transfer rates in the present experiments is based upon a knowledge of the temperature of the water side of the absorbing tube's metal surface.

To determine these metal temperatures, two thermo-couples were used, made especially for the experiments by The Bristol Company. The couples were made of special alloys, the exact composition of which is not available, but it is stated by the manufacturer to be such as to give the highest electro-motive force and at the same time a uniform increase of the same for increasing differences of temperature between the hot and cold ends of the thermo-couples. The couples (see Fig. 8) consisted of two separate parts in the form of the letter C, the hot ends being pointed and wire leads being soldered to each leg of the cold ends. A delicate Weston galvanometer was used to record the deflection.

In order to avoid the complications arising from the use of twenty thermo-couples, the following method of computing metal temperatures was evolved, necessitating the use of but two such couples.

If cooling water is circulated over a hot metal surface, the quantity of heat absorbed per unit area of surface by the water must be a function of the difference in temperature between water and metal and of the rate of water flow. Assuming then that the heating surface of one jacket is the unit of surface, by measuring the water temperatures, the metal temperatures and the quantity of water circulating through a jacket under varied operating conditions, the desired function can be obtained, and from such a relation, any value of the temperature difference between water and metal determined. With the temperature difference and the water temperature known, the metal temperature can be obtained.

A special series of tests was run to determine this relation. The two thermo-couples were used in connection with jackets Nos. 3, 4, 5, 10, 15, 18 and 19. A number of gas weights were used to show a wide variation in the heat absorbed and the rates of

water flow through the jackets were varied from the minimum to the maximum, in this way covering all conditions of the tests.

The thermo-electric couples were carefully calibrated:

First, to verify the setting of the couples on the tubes, that is, to assure a positive electric contact. This calibration was made with a hot end temperature range of from 130 to 150 degrees and with a cold end temperature of from 70 to 90 degrees to correspond to the room temperatures.

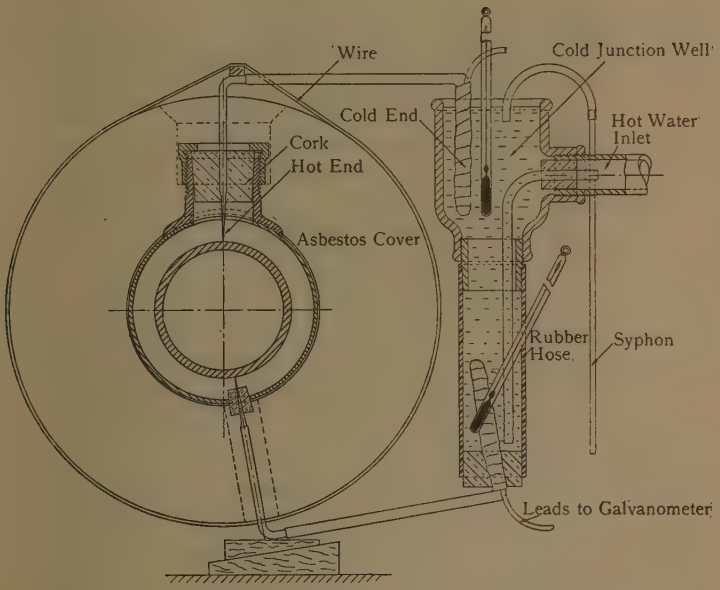


FIG. 8 THERMO-COUPLES IN POSITION AND ARRANGEMENT OF COLD WELL

Second, a calibration covering the range of tube temperatures in the transfer rate tests for the hot end, namely, from 140 to 212 degrees Fahrenheit, and with the cold end range from 130 to 150 degrees Fahrenheit, the latter to cover the range of temperatures of the overhead water supply, the cold end during the couple tests being immersed in water from this supply.

For the method of calibration, see Appendix D.

From the data obtained in these calibrations, curves were drawn for each cold end temperature for each couple, having as ordinates the hot end temperatures and as abscissae the corresponding galvanometer readings. From these plots the following equations were computed, in which

T=hot end temperature degrees Fahrenheit,
t=cold end temperature degrees Fahrenheit,
m=galvanometer reading.

Couple No. 1

Setting range $T=(0.05715-0.000098t)m+1.08t-23.4$ (2-a)
Tube temperature range $T=0.00436m+0.8583t+6.4$ (3-a)

Couple No. 2

Setting range $T=(0.0598-0.00014t)m+1.95t-32.7$ (2-b)
Tube temperature range $T=0.0417m+0.960t-7.4$ (3-b)

With these equations thus determined, the thermo-couple tests were conducted with the apparatus as follows:

Fig. 8 shows the couples in position for determining the temperature at any section of the transfer tube. That portion of the tube on which the points of the couple were to rest was carefully cleaned and polished in order to secure positive electric contact, thus reproducing calibration conditions. This contact was further assured by binding a wire, insulated from the couple around the pipe and over the couple as shown. The cold ends of the couple were immersed in wells filled with water at room temperature, the furnace was kept cold and no suction applied. Hot water was then circulated through the jackets for a period sufficiently long to bring the temperature of the tube to that of the circulating water. Under such conditions the temperature of the metal, measured by means of a thermo-couple, see formula (2-a) or (2-b), must be the same as that of the circulating water and no tests were run for determining tube temperatures under operating conditions until the settings of the couples were thus verified.

As soon as a proper couple setting was assured, the cold end water wells were supplied with water from the overhead tank at temperatures equivalent to those found in the transfer rate experiments. The supply to the water wells flowed in at the bottom and a continual flow was maintained through the wells

by a syphon, thus preventing the cooler water from settling at the bottom of the well and maintaining the proper circulation about the cold ends of the couples. Two thermometers placed next to each cold end of each couple were used to record the water temperatures for both setting and tube range temperature tests.

The exhauster and furnace were then started and test conditions established. No thermo-couple readings were recorded until the apparatus had been in operation for at least an hour. Five series of tests were conducted for the determination of the metal temperature, the amount of circulating water being widely varied for each rate of gas flow, thus covering the range of the transfer experiments. The locations of the couples in these tests were as follows:

1st series, couple No. 1 in jacket No. 4; couple No. 2 in jacket No. 18
2nd series, couple No. 1 in jacket No. 4; couple No. 2 in jacket No. 18
3rd series, couple No. 1 in jacket No. 10; couple No. 2 in jacket No. 15
4th series, couple No. 1 in jacket No. 5; couple No. 2 in jacket No. 19
5th series, couple No. 1 in jacket No. 3; couple No. 2 in jacket No. 10

For this series of thermo-couple tests the weight of water flowing per hour through the jackets tested and the heat absorbed per hour by these jackets were computed as in the transfer rate tests. The temperatures of the metal surfaces of the jackets tested were computed from formulae (*3-a*) and (*3-b*).

In the study of these results for the determination of the relation between the difference in temperature between metal and water, the rate of water flow and the temperature of the water, two assumptions were made which, inasmuch as the dimensions of any individual jacket were small, are warranted.

First, that the difference in temperature between the metal and the water is a constant throughout the length of any jacket; and second, that the temperature of the water throughout the length of one jacket follows a straight line law.

Hence, if T_1=inlet temperature of water to a jacket,

T_2=outlet temperature of water from a jacket,

and t=temperature of metal surface at the center of a jacket, since the metal temperature is measured at the center of a jacket,

Δt, the difference between metal and water temperatures,

$$=t-\frac{T_1+T_2}{2}$$

for any jacket.

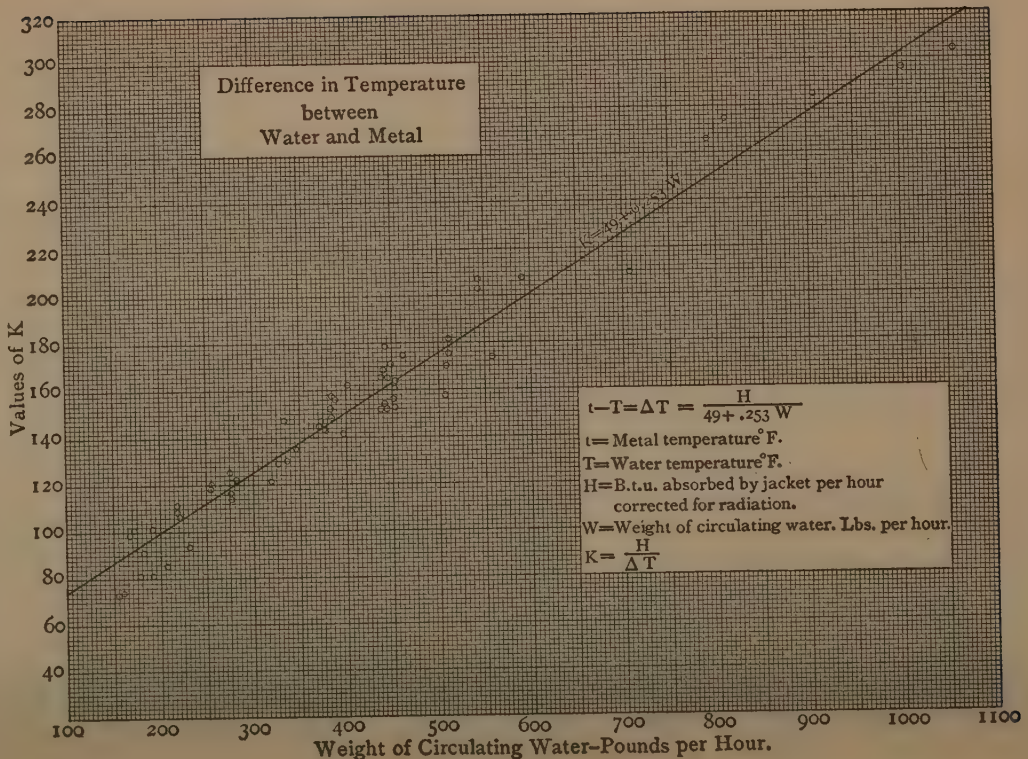

Difference in Temperature
between
Water and Metal
Values of K
320
300
280
260
240
220
200
180
160
140
120
100
80
60
40
100
200
300
400
500
600
700
800
900
1000
1100
Weight of Circulating Water–Pounds per Hour.
t−T=ΔT = H / 49+.253 W
t= Metal temperature °F.
T=Water temperature °F.
H=B.t.u. absorbed by jacket per hour corrected for radiation.
W=Weight of circulating water. Lbs. per hour.
K= H / ΔT

FIG. 9

After a considerable study of the computed data, it was found that the ratio of heat units absorbed by a jacket per hour (corrected for radiation) to the corresponding differences in temperature between metal and water gave a straight line function of the water weights. Fig. 9 gives the computed results plotted in this way. From this plot the formula giving the relation between the temperature difference between water and metal, the heat absorption and the water weights were determined as follows:

$$\frac{H_1}{\Delta T}=49+0.253W$$

or

$$\Delta T=\frac{H_1}{49+0.253W} \qquad (4)$$

where H_1=the B. t. u. absorbed per jacket per hour, corrected for radiation,

ΔT=difference in temperature between metal and circulating water,

W=weight of water circulating through the jacket in pounds per hour.

The metal temperatures at the ends of each jacket were computed from the formula,

$$t_1=T_1+\Delta T=T_1+\frac{H_1}{49+.253W} \qquad (5)$$

In order to determine whether there was any change in the metal temperature at different points of the periphery of the transfer tube on a plane perpendicular to the direction of gas flow, additional thermo-couple tests were run. Fig. 8 shows the manner in which two thermo-couples were placed at the center of jacket No. 3 where the angle between the couples was 170 degrees. These four tests gave the following results:

Test No.	Gas Rate Cubic Feet Per Minute	Metal Temperatures Degrees Fahrenheit Above Pipe	Below Pipe	Difference in Temperature
1	5	192.9	193.0	—0.1
1-a	5	194.3	193.6	+0.7
2	6	204.1	202.6	+1.5
2-a	6	204.9	203.2	+1.7

These figures show that the difference in temperature of different points of the transfer tube on the same plane are such as to give results well below the error of the experiments.

THE DETECTION AND CORRECTION OF ERRORS. CHECKS ON RECORDED DATA AND COMPUTED RESULTS

A RECORD of the water heads in the various discharge tanks served to verify a steady flow into the weighing buckets while readings were taken. Large errors in recording water weights could be detected by means of calibration plots of the orifices used on the tanks where the water heads vary as the square of the weights discharged.

It was found that the greatest source of possible error resulted from a change in the temperature of the water supplied to the jackets and cooler.

From data obtained in different tests, it was found that a molecule of water took approximately one minute to flow through jacket No. 1 and something less than five minutes to flow through jacket No. 20, while it took from 13 to 20 minutes for a molecule of water to pass through the cooler—depending upon the rate of flow. Any change in the temperature of water supply for a given set of gas conditions resulted in a corresponding change in exit water temperatures from the jackets and the cooler. Any one series of readings occupied a period of 6 minutes. Thus while any change in temperature of water supply would have no effect upon the temperature difference between inlet and outlet water temperatures of jackets, it would affect the corresponding difference in the cooler, that is, there would be a lagging effect through the cooler that might occur at a time to affect results.

It was such a change in the entering temperature of the water to the cooler that made it necessary to discard a few tests.

The greatest temperature change in water supply observed during any one day was 3 degrees Fahrenheit. When such a change occurred toward the latter part of any day, it was possible to correct for it as follows.

As has been described, the apparatus was operated throughout the morning to assure steady conditions and the data was observed in the afternoon. Three tests were usually run during each afternoon and the gas conditions during these three tests were perfectly steady. It was when a change in the inlet water

temperature occurred during the last test that a correction was possible.

For a steady condition the weight of gas flowing through the transfer tube was the same for all three tests as was the mean specific heat of the gas through the jackets and cooler. Since the heat absorbed by the cooler must equal the heat given up by the gases, we have three equations, one for each test, as follows:

$$W_1 \ \Delta t_1 = w_1 \ C_1 \ \Delta T_1$$
$$W_2 \ \Delta t_2 = w_2 \ C_2 \ \Delta T_2$$
$$W_3 \ \Delta t_3 = w_3 \ C_3 \ \Delta T_3$$

where 1, 2 and 3 indicate number of test during afternoon,

W = weight of water in pounds per hour through cooler,
w = weight of gas in pounds per hour through cooler,
Δt = rise in water temperature through cooler,
ΔT = drop in gas temperature through cooler,
C = mean specific heat of gases through cooler.

Dividing the third equation by the first and second,

$$\frac{W_3 \ \Delta t_3}{W_1 \ \Delta t_1} = \frac{w_3 \ C_3 \ \Delta T_3}{w_1 \ C_1 \ \Delta T_1}$$

$$\frac{W_3 \ \Delta t_3}{W_2 \ \Delta t_2} = \frac{w_3 \ C_3 \ \Delta t_3}{w_2 \ C_2 \ \Delta t_2}$$

But $w_3 C_3 = w_2 C_2 = w_1 C_1$ (for constant conditions).

Hence
$$\frac{W_3 \ \Delta t_3}{W_1 \ \Delta t_1} = \frac{\Delta T_3}{\Delta T_1}$$

and
$$\frac{W_3 \ \Delta t_3}{W_2 \ \Delta t_2} = \frac{\Delta T_3}{\Delta T_2}$$

or
$$\Delta t_3 = \frac{\Delta T_3}{\Delta T_1} \times \frac{W_1}{W_3} \ \Delta t_1$$

and
$$\Delta t_3 = \frac{\Delta T_3}{\Delta T_2} \times \frac{W_2}{W_3} \ \Delta t_2$$

These equations were solved for Δt_3 from values of the other terms observed in the tests for the day. If these two computed values of Δt_3 agreed, it was compared with the observed value of Δt_3 and such comparison served to check whether the change

in temperature of water supply had a lagging effect on the outlet temperature from the cooler. If this was found to be the case and the outlet temperature from the cooler was questionable, the computed value was used instead of the observed value.

Another source of error lay in the observation of the dew point temperature. Here the personal element of the observer unquestionably played a part. Though an improper focusing of the magnifying glass with which the polished pipe was observed might cause a late observation of the dew point, it was found in most cases where the dew point was incorrectly observed that the observation was early. This might be explained by the settling of particles of dust on the polished surface of the pipe or possibly by the reflection of the light from the lamp by which this observation was made. A few tests were discarded because of the improper dew point observation, the method of detecting such error being as follows:

When complete combustion of a fuel containing hydrogen takes place, the weight of water vapor and of carbon dioxide formed for each pound of fuel is a constant. This is almost regardless of the amount of moisture in the air supplied for combustion since the moisture content of such air is negligible when compared with that formed by the burning of hydrogen. The dew point of the gases of combustion is a function of the percentage of moisture in the gases. The percentage of water vapor and carbon dioxide in the gases depends upon the amount of excess air supplied. The relation between the dew point temperature and the percentage by analysis of carbon dioxide will be the same as the relation between the dew point temperature and the percentage of moisture in the gas. Thus, just as any increase in the percentage of water vapor would be accompanied by an increase in the dew point temperature, so will an increase in the percentage of CO_2 be accompanied by an increase in dew point temperature.

A plot was drawn having as ordinates the observed dew point temperatures from recorded data and as abscissae the corresponding percentages of carbon dioxide. This plot formed a smooth curve from which any appreciable error in the measurement of the dew point temperature could be detected. While the analysis of the illuminating gas was not absolutely constant throughout

the experiments, the variation in the percentage of hydrogen was too small to have an appreciable effect upon this method of correction.

A check on the computed gas weights was given by means of the draft readings through the absorbing tube. The draft losses were recorded in inches of water between jackets Nos. 1 and 7, between Nos. 7 and 13 and between Nos. 13 and 20. By plotting the draft losses as ordinates and the corresponding computed gas weights as abscissae, a smooth curve was obtained in which the draft losses varied as the square of the gas weights. This curve gave a method of detecting errors in the computed gas weights.

The gas weights might have been computed from the fuel and exit gas analyses, together with the fuel consumption, and while this method was not of sufficient accuracy for these experiments, it was used on numerous occasions as a general check on computed gas weights.

When all computations were completed, the results were plotted, as shown in Figs. 10 and 11,* using as ordinates heat transfer rates and as abscissae temperature differences between the hot gases and the metal. Each curve represents a single test and is for a constant weight of gas. A general inspection of these curves indicates that they must fall one above the other without crossing in the order of the gas weights. If any two curves crossed, there was obviously some error either in the original data or in the work of computation. If the error occurred in the gas weights, it was multiplied as many times as there were jackets and the error would be least for the lower temperature difference. Hence, the first point plotted gave the general location of the curve with respect to adjacent curves.

* These are typical of such curves plotted for all tests.

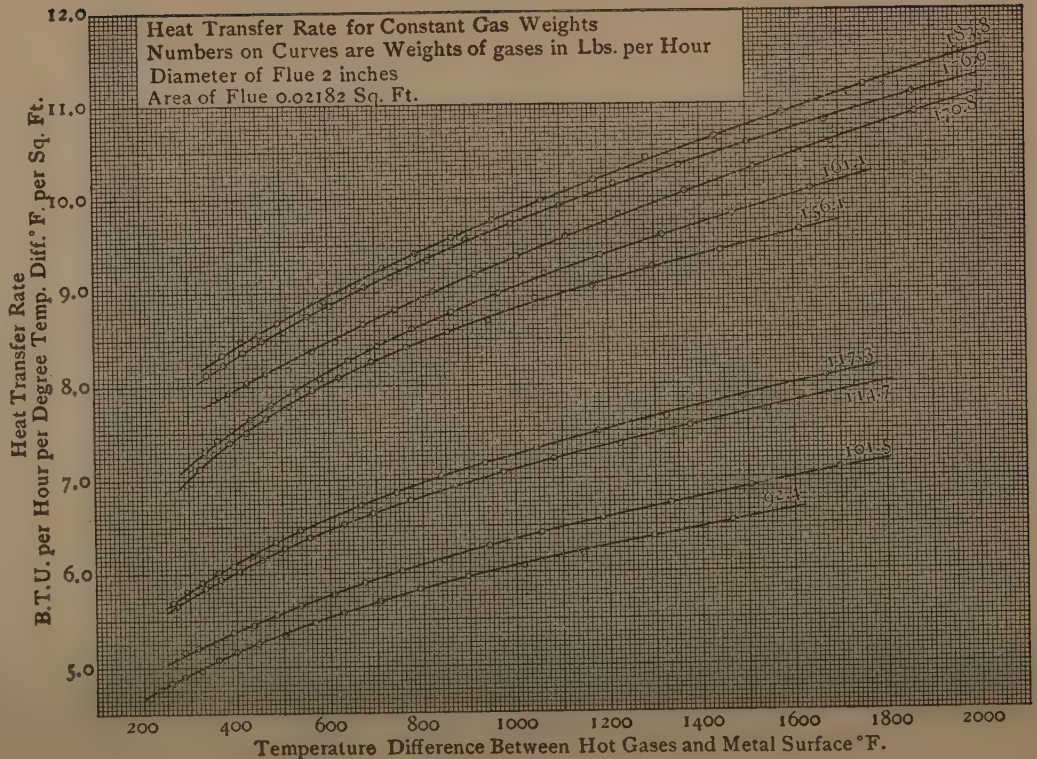

Heat Transfer Rate for Constant Gas Weights
Numbers on Curves are Weights of gases in Lbs. per Hour
Diameter of Flue 2 inches
Area of Flue 0.02182 Sq. Ft.
Heat Transfer Rate
B.T.U. per Hour per Degree Temp. Diff.° F. per Sq. Ft.
12.0
11.0
10.0
9.0
8.0
7.0
6.0
5.0
200
400
600
800
1000
1200
1400
1600
1800
2000
Temperature Difference Between Hot Gases and Metal Surface °F.
192.5
161.3
156.1
173.3
174.7
191.5
192.1

FIG. 10

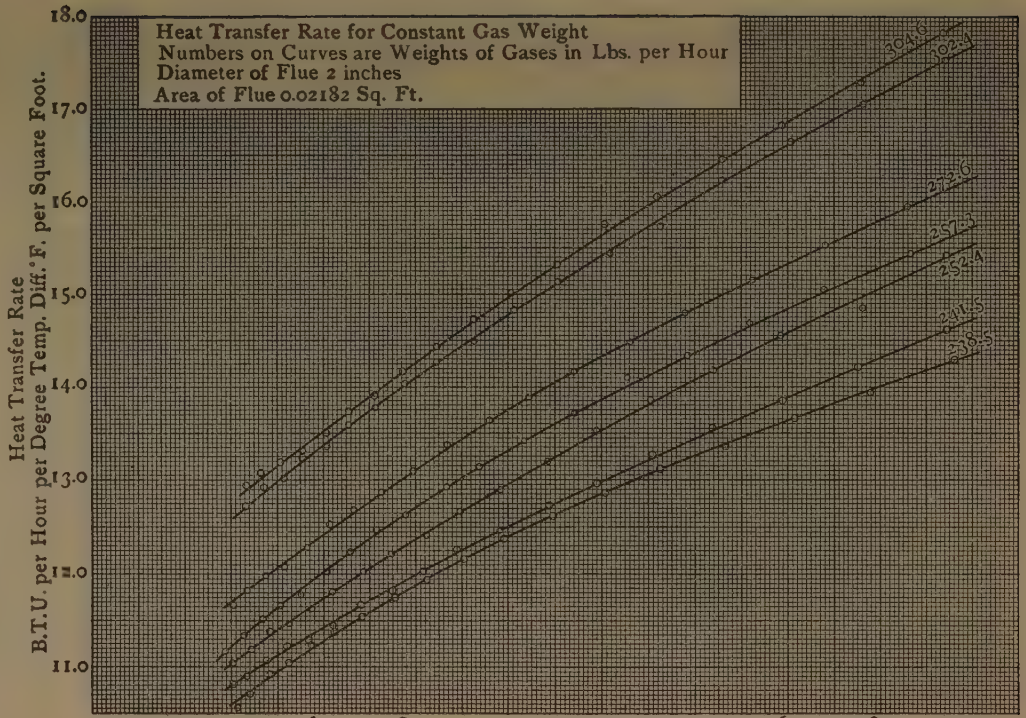

Heat Transfer Rate for Constant Gas Weight
Numbers on Curves are Weights of Gases in Lbs. per Hour
Diameter of Flue 2 inches
Area of Flue 0.02182 Sq. Ft.
304.0
302.3
272.6
257.1
252.1
240.5
238.5
Heat Transfer Rate
B.T.U. per Hour per Degree Temp. Diff. °F. per Square Foot.
18.0
17.0
16.0
15.0
14.0
13.0
12.0
11.0
200
400
600
800
1000
1200
1400
1600
1800
2000
Temperature Difference Between Hot Gases and Metal Surface° F.

FIG. 11

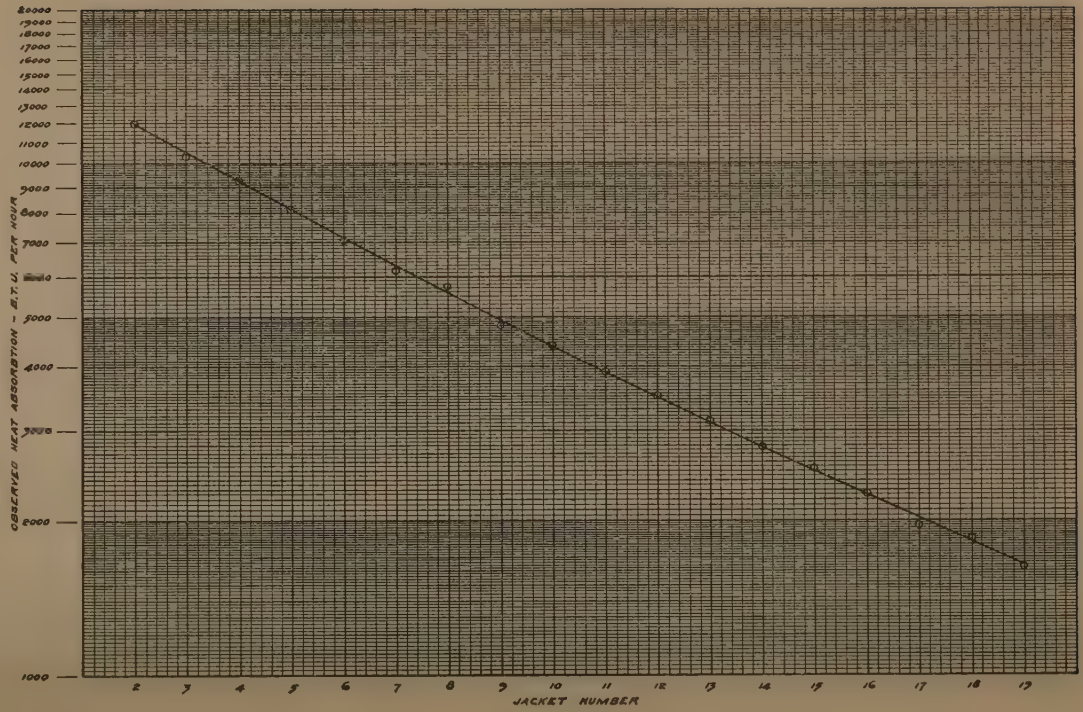

OBSERVED HEAT ABSORPTION - B.T.U. PER HOUR
20000
19000
18000
17000
16000
15000
14000
13000
12000
11000
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
JACKET NUMBER

FIG. A

METHODS USED IN MAKING COMPUTATIONS

BEFORE describing in detail the methods of calculation, a discussion of two of the phases of computation is perhaps desirable for the sake of clarity. These are: first, the use of a corrective formula to be applied to the observed heat absorption of different jackets; and second, a study of the final formula used in calculating heat transfer rates.

Corrective Formula for Heat Absorption

If a plot is made with the heat absorbed by each of the jackets in the experimental apparatus as ordinates and the number of the jacket as abscissae, it should be possible under ideal conditions to draw a smooth curve through all points. Ideal conditions would demand that there was no error made in determining the temperature rise of the water through the jackets and that the weight flowing in any given time was correctly determined. Further, it would assume that either the temperature of the tube exposed to the gas would be constant or else vary uniformly from one end to the other. This latter condition was taken care of by adjusting the amount of water that was passing through the various jackets in such a way that it gradually increased from one end to the other. It is true that it was taken care of only approximately, but the error introduced in this way is small as compared with accidental errors made in the observations. The errors made in observations when the rate of flow of water varies, as it would if the orifice became slightly fouled or if any one of a number of other things should happen, would affect the heat absorbed in that jacket by throwing off the point from the uniform curve stipulated as representing ideal conditions. It is, therefore, to be expected, that an actual plot would show points, some on one side and some on the other of this ideal curve. If a method could be devised to locate the points corresponding to the ideal curve, it would serve as a means for correcting for some of the errors. For instance, if it was known that all the points were on a horizontal straight line, then an average would be considered as representing the value nearest determined by the test, and

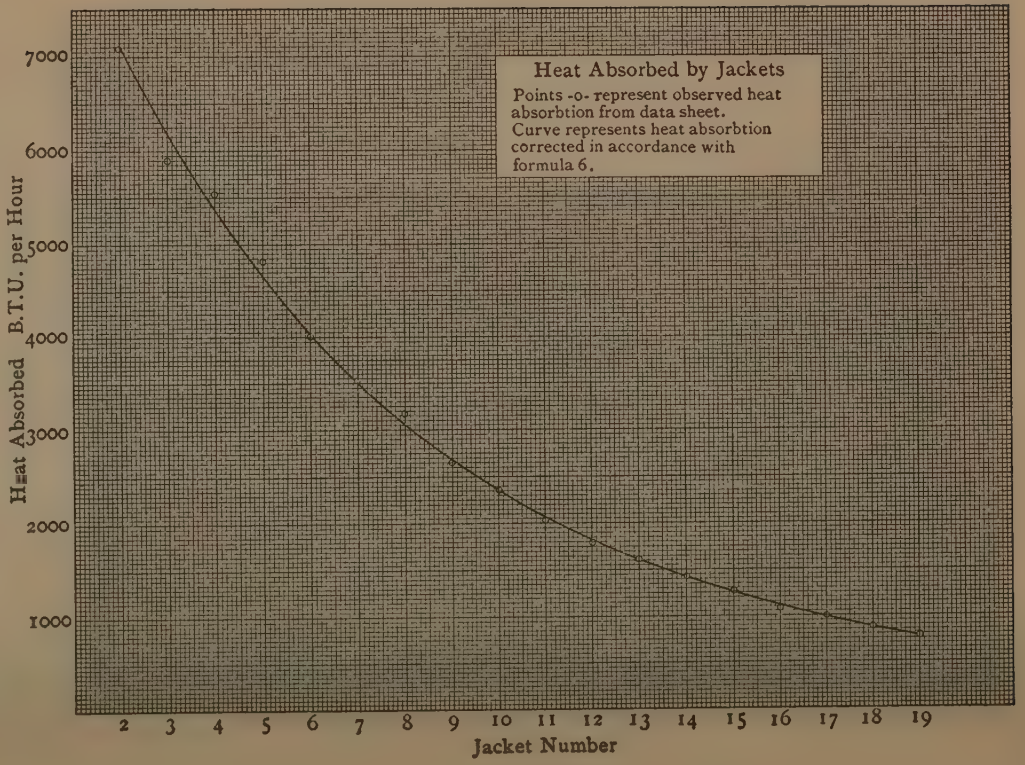

Heat Absorbed by Jackets
Points -o- represent observed heat absorbtion from data sheet.
Curve represents heat absorbtion corrected in accordance with formula 6.
Heat Absorbed B.T.U. per Hour
7000
6000
5000
4000
3000
2000
1000
2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
Jacket Number

FIG. B

without hesitation any observer would take an average in this way and use it. If it was known that the points fell on a straight line that was inclined to the axis, then the method of least squares gives a simple way of determining what this line should be and warrants the use of measurements corrected in this way.

It so happens in connection with the heat transfer experiments that the form of the smooth curve representing the heat absorbed by the jackets is not known. It was found, however, that if the logarithms were plotted against the jacket numbers, the points would fall very nearly on a straight line, the line being inclined of course to the axis. Fig. A shows a typical example of such a plot, the logarithms of the observed heat absorption per jacket being plotted against the jacket number. If, therefore, it was assumed that the equation of this line could be represented by a curve of the second degree, the coefficient of the term involving the square would be small but nevertheless would correct the results to conform to what experiments in general indicate they should be. In working up the results consequently, it was assumed that the smooth curve could be expressed by such an equation. The coefficients were determined so that they had the most probable value and in the calculations later, the heat absorbed by the jackets was taken from these corrected values.

Thus, if H_x=the heat absorbed in one jacket per hour without radiation correction,

and x=the number of jacket beginning at the furnace end,

the heat absorbed by any jacket may be expressed, in terms of its log, as

$$\log_{10} H_x = a + bx + cx^2 \qquad (6)$$

where a, b and c are constants to be determined by the method of least squares.

Fig. B shows the relation between the heat absorption per jacket as corrected by this method and the actual observed absorption. The curve is drawn from the corrected absorption values while the points represent the observed absorption per jacket in the particular test that is later considered in detail. This curve is representative of a complete series drawn in this manner.

In the present determinations the first and last jackets are not included; the first because of its proximity to the furnace and the last because there is no adjacent jacket on the side away from the furnace.

For the method of deriving values of a, b and c, see Appendix A.

Transfer Rate Formula

If the rate of cooling of the gases through any one jacket is uniform and the metal temperature throughout that jacket is constant, it would be possible to express the transfer rate for any jacket by the formula

$$R = \frac{H}{(T_m - t_m)S} \qquad (7)$$

where R = the transfer rate,
H = the heat absorbed,
T_m = the mean temperature of the gases,
t_m = the mean temperature of the metal,
and S = the surface of the jacket in square feet.

Under such conditions the mean temperature of the gases would be the average temperature. If the temperature of the metal varied uniformly throughout the length of an individual jacket, and because such length is relatively small such an assumption would seem warranted, then the mean temperature of the metal would be the average temperature. The rate of cooling of the gases, however, is not uniform, and formula (7) can only represent an approximate value of the rate of heat transfer.

Since the rate of cooling of the gases is not uniform, it is necessary to derive a formula which will take into consideration the variation in such rate. Such a formula is derived in Appendix B and is used in computing the final transfer rates. This formula is as follows:

$$R = \frac{wc}{S} \log_e \left(\frac{T_1 - t_1 - (t_1 - t_2)\frac{wc}{RS}}{T_2 - t_2 - (t_1 - t_2)\frac{wc}{RS}} \right) \qquad (8)$$

where R = the heat transfer rate in B. t. u. per hour per one degree difference in temperature per square foot of heating surface,

t_1=temperature of metal at hot end of jacket,
t_2=temperature of metal at cold end of jacket,
T_1=gas temperature at hot end of jacket,
T_2=gas temperature at cold end of jacket,
w=weight of gases flowing through pipe in pounds per hour,
c=specific heat of gases,
and S=surface of jacket in square feet.

Since formula (*8*) is a transcendental equation in that it cannot be solved for R, a method of approximation must be used, as follows:

A trial value of R is determined from formula (*7*) in which $(T_m - t_m)$ is the average temperature difference between the metal and the gases. This trial value is substituted in the right hand member of equation (*8*) and the equation solved for R. Such a solution gives a second approximation for the true value of R, which is again substituted in the right hand member of equation (*8*) and a third value found. This method was followed until there was no variation for three decimal places in the last approximation and the final value.

The calculations involved in the final determination of these experiments extended over a period of three months. Each figure was computed by one man and re-computed by a second, thus furnishing a check on every result.

The methods of computation are probably best indicated by an example, and for this purpose the third test of March 17 has been chosen. The test data for this run is given on a facsimile test sheet, Table 1, page 50, and the methods of calculation are as follows:

The time of the test indicates the period during which water was allowed to flow into the weighing buckets from the jackets and the cooler. The atmospheric pressure in pounds per square inch was found from the barometric readings and a curve of mercury weights plotted from values as given in the Smithsonian Tables. In the test in question this pressure is 14.67 and 14.68 pounds per square inch.

The relative humidity is obtained from the psychometric tables published by the Weather Bureau, U. S. Department of Agriculture, Bulletin No. 235 (1912). The fuel gas analysis

HEAT TRANSFER EXPERIMENTS

NO. OF TEST 3 — TIME OF TEST 3:38 TO 4:06 — THE BABCOCK & WILCOX CO. — DATE MAR. 17, 1914. — WEATHER FAIR

BAROMETER - INS. Hg.	30.008	30.018
BAROMETER - LBS. SQ. IN.	14.67	14.68
BAROMETER TEMP. - °F.	80	80
ROOM TEMPERATURE - °F.	82.5	82.0
WET BULB TEMP. - °F.	62.5	61.6
DRY BULB TEMP. - °F.	76.5	75.7
HUMIDITY - PER CENT	44.5	44.0
GAS PRESSURE - INS H_2O	3.20	3.20
AIR PRESSURE - INS. Hg	2.00	2.00

FUEL GAS FLOW	
METER AT 3:45 PM - CU. FT.	379694
METER AT 4:15 PM. - CU. FT.	379795
FLOW PER MIN. - CU. FT	3.37
FLOW PER HOUR - CU. FT	202

FUEL GAS ANALYSIS	% BY VOL	% BY WT.
CO_2	3.9	9.09
O_2	.0	.00
CO	23.2	34.42
C_2H_6	6.8	10.09
CH_4	33.6	28.49
H_2	22.0	2.33
N_2 (DIFF)	10.5	15.58
TOTAL	100.0	100.00
B.T.U. PER CU. FT (CALC.)		625.29

DRAFT		
NO. 1 TO ATMOSPHERE - INS	.56	.56
NO. 1 TO NO. 7 - INS.	.055	.05
NO. 7 TO NO. 13 - INS.	.08	.075
NO. 13 TO NO. 20 - INS.	.095	.09
TOTAL	.790	.775
NO. 20 TO ATMOSPHERE - INS	.77	.76

COOLER	
GAS TEMPERATURE IN - °F.	388
GAS TEMPERATURE OUT - °F.	166.5
WATER TEMPERATURE IN - °F	127.55
WATER TEMPERATURE OUT - °F.	158.3

FLUE GAS ANALYSIS	% BY VOL
CO_2	11.0
O_2	6.9
CO	0.0
N_2 (DIFF.)	82.0
	100.0

DEW POINT APPARATUS	
WATER INLET TEMP. - °F	122.5
WATER OUTLET TEMP. - °F.	123.5
DEW POINT OF FLUE GAS - °F.	123.0
DIS. BURNER PLATE TO FURNACE - IN	2
H S PER JACKET - SQ. FT.	.5236

FLOW OF WATER — BOOK 18 — TIME 6 MIN.

TANK NO.	DIAM. ORIFICE	TOTAL WT. LBS.	TOTAL WT. OZ.	TARE LBS.	TARE OZ.	NET WT. LBS.	NET WT. OZ.	POUNDS PER HOUR	BOOK 13 HEAD FT	BOOK 17 HEAD FT.	BOOK 13 HEAD FT.	BOOK 17 HEAD FT.
1	3/8	157	7	72	0¼	85	6¼	854.06	1.240	1.240	1.237	1.235
2	11/32	78	2	5	7¼	73	1¼	731.09	1.048	1.045	1.044	1.044
3	5/16	83	0	5	7¼	79	8¼	795.31	1.732	1.731	1.730	1.730
4	5/16	80	7½	5	7¾	74	15¾	743.84	2.142	2.141	2.145	2.145
5	5/16	75	7½	5	7	70	0½	700.31	1.905	1.903	1.900	1.900
6	1/2	61	8	5	8¼	55	15½	559.69	1.944	1.940	1.934	1.931
7	1/4	62	6½	5	2¼	56	12¼	567.97	2.047	2.046	2.045	2.042
8	1/4	56	1¾	5	6	50	11¾	507.34	1.745	1.745	1.745	1.745
9	7/32	49	15¾	5	12	44	3¾	448.34	2.005	2.002	1.995	1.992
10	7/32	46	3¼	5	8¼	40	11	406.88	1.761	1.760	1.754	1.751
11	7/32	45	3¼	5	1¼	40	2	401.25	1.695	1.693	1.691	1.693
12	7/32	40	6	5	5½	34	16½	347.81	1.277	1.280	1.268	1.270
13	7/16	38	14¾	5	6	33	8¾	335.47	2.346	2.400	2.240	2.250
14	7/16	35	6¼	5	6	30	0¼	300.47	1.650	1.650	1.618	1.630
15	7/16	36	9	5	6¼	31	2¾	311.72	1.845	1.843	1.846	1.843
16	7/16	31	3½	5	6¼	25	13¼	238.29	1.250	1.248	1.236	1.240
17	9/32	25	13¼	5	6¼	20	6¾	204.06	1.627	1.642	1.544	1.533
18	9/32	25	3	5	6¼	19	18¼	197.81	1.540	1.543	1.525	1.526
19	9/32	26	7	5	4¾	21	2¼	211.41	1.742	1.745	1.732	1.733
20	9/32	25	13	4	12¾	20	14¼	208.91	1.722	1.736	1.700	1.704
21	1/8	62	9¼	5	11¾	56	13¼	568.59	2.060	2.060	2.060	2.060

OUTLET WATER TEMPERATURES						INLET WATER TEMPERATURES								
	BOOK NO. 13	BOOK NO 17	BOOK NO 13	BOOK NO 17			BOOK NO 14	BOOK NO 15	BOOK NO 14	BOOK NO 15				
JACKET NO	OUTLET TEMP. - °F.	OUTLET TEMP. - °F.	OUTLET TEMP. - °F	OUTLET TEMP. - °F.	AVERAGE TEMP. - °F.	JACKET NO.	INLET TEMP. - °F	INLET TEMP. - °F	INLET TEMP. - °F	INLET TEMP. - °F	AVERAGE TEMP. - °F.	TEMP. DIFF. - °F.	HEAT ABSORBTION B.T.U. PER HOUR	CORRECTED HEAT ABSORBTION.
1	158.3	158.3	158.3	158.3	158.3	1	148.0	148.0	148.0	147.9	148.0	10.3	9826.82	
2	157.5	157.7	157.6	157.5	157.6	2	147.8	148.0	148.0	148.0	147.95	9.7	7091.57	7140.1
3	155.4	155.4	155.4	155.4	155.4	3	147.9	148.0	148.0	148.0	148.0	7.4	5885.29	6180.8
4	155.3	155.4	155.4	155.4	155.4	4	147.9	148.0	148.1	148.1	148.0	7.4	5548.82	5359.6
5	154.7	154.7	154.8	155.0	154.8	5	147.8	147.9	148.0	148.1	147.95	6.9	4832.14	4655.3
6	155.0	155.1	155.2	155.1	155.1	6	147.7	148.0	147.9	147.9	147.9	7.2	4029.82	4050.6
7	154.2	154.2	154.2	154.3	154.2	7	147.9	148.0	148.0	148.0	148.0	6.2	3217.31	3230.7
8	154.2	154.1	154.2	154.2	154.2	8	148.0	148.0	148.0	147.8	147.95	6.3	3126.24	3088.2
9	153.7	153.9	154.0	154.0	153.9	9	147.7	148.0	148.0	147.8	147.9	6.0	2654.40	2695.5
10	153.7	153.7	153.8	153.7	153.7	10	147.8	147.9	148.0	147.8	147.9	5.8	2329.90	2361.4
11	153.0	152.9	153.0	152.9	152.95	11	147.9	147.9	148.0	147.9	147.9	5.1	2046.38	2078.8
12	153.0	153.0	153.0	152.8	152.95	12	147.0	147.9	147.9	147.8	147.85	5.1	1773.83	1821.3
13	152.7	152.7	152.6	152.5	152.6	13	147.9	147.9	147.8	147.7	147.8	4.8	1610.26	1603.9
14	152.5	152.4	152.4	152.3	152.4	14	147.7	147.7	147.7	147.6	147.7	4.7	1412.21	1414.7
15	151.8	151.7	151.9	151.8	151.8	15	147.9	147.7	147.8	147.8	147.8	4.0	1246.88	1249.9
16	151.8	151.9	151.8	151.8	151.8	16	147.6	147.7	147.5	147.6	147.6	4.2	1084.78	1106.2
17	152.0	152.0	152.0	152.0	152.0	17	147.3	147.2	147.2	147.3	147.25	4.8	979.49	980.7
18	151.7	151.7	151.8	151.7	151.7	18	147.3	147.3	147.3	147.3	147.3	4.4	870.76	871.0
19	150.9	150.8	150.9	150.9	150.9	19	147.3	147.2	147.2	147.3	147.25	3.7	782.22	774.8
EO	150.9	150.9	150.9	150.9	150.9	20	147.1	147.2	147.2	147.2	147.2	3.7	772.97	
COOLER	138.4	138.4	138.3	138.2	138.3	COOLER	147.5	147.5	147.6	147.6	147.55	10.8	6140.77	

TABLE 1

was made in the laboratory of The Babcock & Wilcox Co., the volumetric percentages of the various constituents being determined and the percentages by weight being found in the ordinary way.

The flue gas analysis is the average of those taken before and after the period of water flow.

The weights of water per hour as given are the weights flowing during the 6-minute period of flow multiplied by 10.

The heat absorbed by each jacket is determined by multiplying the weight of water per hour by the difference between the average inlet and outlet water temperature.

It has been stated that the corrected value of the heat absorbed by a jacket is expressed in terms of its log by

$$\log_{10} H_x = a + bx = cx^2 \qquad (6)$$

For the present test, with the values of a, b and c determined after the method given in Appendix A, formula (6) takes the form

$$\log_{10} H_x = 3.91709884 - 0.0637667x + 0.00036999x^2$$

Since, as stated, jackets Nos. 1 and 20 are not considered, jacket No. 2 is taken as No. 1 and jacket No. 19 as No. 18, the intermediate jackets being given corresponding numbers one lower than their actual sequence number. Substituting then for x all values between 1 and 18, the corrected B. t. u. absorbed by jackets Nos. 2 to 19 are determined, as given in the column headed "Corrected B. t. u.," Table 1.

Determination of Gas Weights

The proportional parts by weight of the constituents of the gases passing through the apparatus are determined from a consideration of the volumetric analysis, the dew point temperature, and the pressure of the gases in the dew point box.

If we consider a cubic foot of gas passing through the dew point box, it is obvious that the total pressure exerted by the gas will be the sum of the pressures exerted by the constituent parts. The water content of the gas is in the form of vapor and as such must exert a pressure. The weight of the water vapor content of the gas per cubic foot can be determined from the dew point temperature, for at this temperature, or that at which the vapor

will start to condense, the gas must be saturated, and the water vapor have a definite density.

If the water vapor in one cubic foot of gas is condensed, its volume is negligible (less than one-tenth of one per cent), and the volume of the dry gas may still be considered one cubic foot. With the vapor condensed, however, the pressure exerted by the dry gas will be diminished by the amount of that exerted by the water vapor.

Considering as dry, then, the gas as volumetrically analyzed, the percentages by weight of the different constituents may be determined in the ordinary way at the dew point temperature and at the pressure diminished by the amount exerted by the water vapor. Adding to the weights so found the weight of water condensed, that is, the density at the dew point temperature, the weight of a cubic foot of the gas is obtained, and finally the proportionate parts by weight (values which are required for the determination of the specific heat of the gas) by dividing the weights of various constituents of the gas by this total weight.

Inasmuch as one inch of draft suction at the dew point box would correspond to a pressure of only 0.036 pounds per square inch below atmospheric pressure, the dew point box pressure has been assumed as atmospheric. The draft suction at this point in the test under consideration was something less than one inch. For the present test,

Dew point temperature=123 degrees Fahrenheit
Vapor pressure at 123 degrees Fahrenheit=1.835 pounds per square inch. (Marks and Davis)
Density pounds per cubic feet=0.00532.

Flue gas analysis,

CO_2=11.1 per cent
O_2= 6.9 per cent
CO = 0.0 per cent
N_2=82.0 per cent (by difference)

For a perfect gas,

$$\frac{P_0 V_0}{T_0} = \frac{P_1 V_1}{T_1} \text{ or } \frac{1}{V_1} = \frac{1}{V_0} \times \frac{P_1}{P_0} \times \frac{T_0}{T_1}$$

Since $\frac{1}{V} = w$ = weight per cubic foot,

$$w_1 = w_0 \times \frac{P_1}{P_0} \times \frac{T_0}{T_1}$$

If n = per cent by volume of a constituent gas,

$$w_1 n = w_0 \times \frac{P_1}{P_0} \times \frac{T_0}{T_1} \times n \qquad (9)$$

where $w_1 n$ = weight of a constituent of the gas in one cubic foot whose percentage by volume is n,

w_0 = density of gas in pounds per cubic foot under normal conditions (*i. e.*, $T_0 = 492$ degrees Fahrenheit and $P_0 = 14.7$ pounds per cubic foot),

T_1 = absolute dew point temperature,

P_1 = atmospheric pressure less water vapor pressure.

For this test then,

$$w_1 n = w_0 \times \frac{14.67 - 1.84}{14.70} \times \frac{460 + 32}{460 + 123} \times n$$
$$= 0.7366 \times w_0 n$$

Values of w_0 have been taken from the Smithsonian Tables as follows:

For CO_2 $w_0 = 0.12269$
O_2 $w_0 = 0.08922$
N_2 $w_0 = 0.07829$

Hence a cubic foot of the gas at the dew point temperature contains (*i. e.*, $w_1 n$ equals)

$CO_2 = .01003$ pounds
$O_2 = .00454$ pounds
$N_2 = .04729$ pounds
$H_2O = .00532$ pounds
.06718 pounds

and the proportionate parts by weights are,

$CO_2 = 14.9$ per cent
$O_2 = 6.8$ per cent
$N_2 = 70.4$ per cent
$H_2O = 7.9$ per cent

These values are used in the determination of the mean specific heat of the gases at constant volume through the cooler.

The instantaneous specific heat of a gas at constant pressure and temperature, t, is expressed by the formula*

$$C_p = a + bt + ct^2 + dt^3 \qquad (10)$$

(The value of the fourth term is so small as to be negligible and has been discarded in the present computations.)

The values of these constants for the various constituents of the gas in question give formula (*10*) the form,
For carbon dioxide,

$$C_p = 0.1983 + 835 \times 10^{-7} t - 16.7 \times 10^{-9} t^2 \qquad (11)$$

For oxygen,

$$c_p = 0.2154 + 0.000019\, t \qquad (12)$$

For nitrogen,

$$c_p = 0.2343 + 0.000021\, t \qquad (13)$$

For water vapor at one pound absolute pressure,

$$c_p = 0.4541 + 32 \times 10^{-7} t + 2825 \times 10^{-11} t^2 \qquad (14)$$

If formula (*10*) represents the instantaneous specific heat, the mean specific heat between the temperatures t_1 and t_2 is expressed by

$$C_{t_1-t_2} = a + \frac{b}{2}(t_2+t_1) + \frac{c}{3}\left((t_2+t_1)^2 - t_2 t_1\right) + \frac{d}{4}(t_2+t_1)\left((t_2+t_1)^2 - 2t_2t_1\right) \qquad (15)$$

where a, b, c and d have the same values for various gases as in (*11, 12, 13* and *14*).

In the present test the inlet gas temperature to the cooler, t_1, is 388 degrees Fahrenheit and the outlet temperature, t_2, is 167 degrees Fahrenheit. Substituting these values of t, and the proper values of the constants a, b, and c for the different constituent gases in formula (*15*), we have the values of the mean specific heats,

$$CO_2 = .2202$$
$$O_2 = .2207$$
$$N_2 = .2401$$
$$H_2O = .4575$$

* See Appendix C, for a consideration of Specific Heat of Gases.

and the mean specific heat of the gas as a whole,

CO_2	0.149 x .2202	= 0.0329
O_2	0.068 x .2207	= 0.0149
N_2	0.704 x .2401	= 0.1690
H_2O	0.079 x .4575	= 0.0361
Mean specific heat		$= c_p = 0.2529$

The heat absorbed by the cooler and the drop in gas temperature through the cooler are known. Equating the heat given out by the gas to that absorbed by the water, we have

$$H = wc_p(t_2 - t_1) \quad \text{or} \quad w = \frac{H}{c_p(t_2 - t_1)} \qquad (16)$$

where H = heat absorbed by the cooler per hour, corrected for radiation,

t_1 and t_2 = inlet and outlet gas temperatures,

c_p = mean specific heat of gas between t_2 and t_1,

w = weight of gas in pounds per hour.

The heat absorbed by the cooler in B. t. u. per hour from observed data is 6141 B. t. u. Corrected for radiation,

$$H = 6141 + 267 = 6408$$

Therefore, (*16*) becomes

$$w = \frac{6408}{0.2529\,(388 - 167)} = 114.7 \text{ pounds}$$

Computation of Gas Temperatures Throughout Transfer Tube

In the jackets, as in the case of the cooler, the heat absorbed by the water in any jacket must equal the heat given up by the gases in their passage through that jacket, or

$$H = wc_p(t_2 - t_1)$$

or

$$t_2 - t_1 = \frac{H}{wc_p} \qquad (17)$$

where t_2 = the temperature of gas entering a jacket,

t_1 = the temperature of gas leaving a jacket,

c_p = the mean specific heat of the gas through that jacket.

As stated, only two gas temperatures were observed in these experiments, those entering and those leaving the cooler, but the temperature of the gas entering the cooler is that leaving jacket

No. 20 and the results are computed in sequence from this jacket toward the furnace.

Since the specific heat of the gas varies with the temperature, the mean specific heat through any one jacket could not be determined without a knowledge of both the entering and exit

TABLE 2

Jacket Number	B. t. u. Absorbed per Hour Corrected for Radiation	Temperature of Gases leaving Jacket	Instantaneous Specific Heat of Gases at Temperature leaving Jacket	Drop in Gas Temperature through Jacket	Temperature of Gases entering Jacket
20	804	388	.2565	27.3	415.3
19	806	415.3	.2572	27.3	442.6
18	902	442.6	.2580	30.5	473.1
17	1012	473.1	.2590	34.2	507.3
16	1137	507.3	.2598	38.2	545.5
15	1281	545.5	.2610	42.8	588.3
14	1446	588.3	.2622	48.0	636.3
13	1635	636.3	.2636	54.2	690.5
12	1853	690.5	.2651	61.0	751.5
11	2103	751.5	.2669	68.8	820.3
10	2392	820.3	.2690	77.8	898.1
9	2727	898.1	.2711	88.0	986.1
8	3113	986.1	.2737	99.1	1085.2
7	3561	1085.2	.2764	112.3	1197.5
6	4082	1197.5	.2797	127.2	1324.7
5	4686	1324.7	.2835	144.1	1468.8
4	5391	1468.8	.2875	163.5	1632.3
3	6212	1632.3	.2923	185.3	1817.6
2	7171	1817.6	.2975	210.2	2027.8

Flue gas analysis by weight:
CO_2 = 14.9 per cent.
O_2 = 6.8 per cent.
CO = .0 per cent.
N_2 = 70.4 per cent.
H_2O = 7.9 per cent.

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Gas weight = 114.7 pounds per hour

temperatures. The drop in gas temperature through any jacket, however, is small compared with the temperature itself and hence the instantaneous specific heat of the gases at the temperature leaving any jacket can differ by only a negligible amount from the actual mean specific heat through that jacket. Thus the instantaneous specific heat of the gases at the temperature leaving

any jacket is considered the mean specific heat of the gases throughout that jacket.

For jacket No. 20, then,

B. t. u. absorbed per hour	=773
Radiation correction	= 31
	H=804
Weight of gas, pounds, per hour	=114.7
Gas temperature leaving jacket	=388 degrees
Instantaneous specific heat, obtained from (*11, 12, 13* and *14*) and the proportionate parts by weight of the gas	= 0.2565

Substituting these values in (*17*),

$$t_2 - t_1 = \frac{804}{114.7 \times .2565} = 27.3 \text{ degrees Fahrenheit}$$

Exit gas temperature	$=t_1=388$ degrees
Drop in gas temperature	$=(t_2-t_1)=27.3$ degrees
Entering gas temperature	$=t_2=415.3$ degrees

This value of t_2, the entering gas temperature to jacket No. 20, is then taken as the exit gas temperature from jacket No. 19, the operation repeated to determine the entering temperature to jacket No. 19 or the exit temperature from No. 18. This method is followed through jacket No. 2.

Table 2, page 56, gives the general arrangement of this portion of the work for any individual test. The B. t. u. absorbed per hour, column 2, are the corrected values taken from the data sheet, page 50, with the radiation correction of 31 B. t. u. added. The temperature of the gases leaving jacket No. 20 is that entering the cooler. The other values are computed as has been described.

Computation of Heat Transfer Rates

At the beginning of this article the final transfer formula rate is given as,

$$R = \frac{wc}{S} \log_e \left(\frac{T_1 - t_1 - (t_1 - t_2)\frac{wc}{RS}}{T_2 - t_2 - (t_1 - t_2)\frac{wc}{RS}} \right) \qquad (8)$$

TABLE 3

Jacket Number	B. t. u. Absorbed per Hour Corrected for Radiation H	Weight of Circulating Water Pounds per Hour W	Temperature Difference between Metal and Water — °F $\Delta T = \frac{H}{49 + .253W}$	Water Temperatures °F		Metal Temperatures °F		Gas Temperatures °F		Temperature Difference between Metal and Gas — °F		Average Temperature Difference between Metal and Water* °F	Average Specific Heat c	Heat Transfer Rate (Approximate) R	Heat Transfer Rate (Accurate) R
				Hot End of Jacket τ_1	Cold End of Jacket τ_2	Hot End of Jacket $t_1 = \tau_1 + \Delta T$	Cold End of Jacket $t_2 = \tau_2 + \Delta T$	Hot End of Jacket T_1	Cold End of Jacket T_2	Hot End of Jacket $T_1 - t_1$	Cold End of Jacket $T_2 - t_2$				
2	7171.1	731.09	30.6	157.6	148.0	188.2	178.6	2027.8	1817.6	1839.6	1639.0	1739.3	.3005	7.87	7.97
3	6211.8	795.31	24.8	148.0	155.4	172.8	180.2	1817.6	1632.3	1644.8	1452.1	1548.5	.2949	7.66	7.74
4	5390.6	749.84	22.6	155.4	148.0	178.0	170.6	1632.3	1468.8	1454.3	1298.2	1376.3	.2899	7.48	7.56
5	4686.3	700.31	20.7	148.0	154.8	168.7	175.5	1468.8	1324.7	1300.1	1142.9	1224.7	.2855	7.31	7.36
6	4081.6	559.69	21.4	155.1	147.9	176.5	169.3	1324.7	1197.5	1148.2	1028.2	1088.2	.2816	7.16	7.22
7	3561.3	567.97	18.5	148.0	154.2	166.5	172.7	1197.5	1085.2	1031.0	912.5	971.8	.2781	7.00	7.05
8	3113.2	507.34	17.5	154.2	148.0	171.7	165.5	1085.2	986.1	913.5	820.6	867.1	.2751	6.86	6.90
9	2726.5	442.34	16.9	147.9	153.9	164.8	170.8	986.1	898.1	821.3	727.3	774.3	.2724	6.73	6.79
10	2392.5	406.88	15.7	153.7	147.9	169.4	163.6	898.1	820.3	728.7	656.7	692.7	.2701	6.60	6.65
11	2103.2	401.25	14.0	147.9	153.0	161.9	167.0	820.3	751.7	658.4	584.5	621.5	.2680	6.46	6.51
12	1852.5	347.81	13.5	153.0	147.9	166.5	161.4	751.7	690.5	585.0	529.1	557.1	.2660	6.35	6.39
13	1634.9	335.47	12.2	147.8	152.6	160.0	164.8	690.5	636.3	530.5	471.5	501.0	.2644	6.23	6.27
14	1445.7	300.47	11.6	152.4	147.7	164.0	159.3	636.3	588.3	472.3	429.0	450.6	.2629	6.13	6.14
15	1280.9	311.72	10.0	147.8	151.8	157.8	161.8	588.3	545.5	430.5	383.7	407.1	.2616	6.01	6.03
16	1137.2	258.28	9.9	151.8	147.6	161.7	157.5	545.5	507.3	383.8	349.8	366.8	.2604	5.92	5.95
17	1011.7	204.06	10.1	147.3	152.0	157.4	162.1	507.3	473.7	349.9	311.0	330.5	.2594	5.58	5.87
18	902.0	197.81	9.1	151.7	147.3	160.8	156.4	473.7	442.6	312.3	286.2	299.3	.2585	5.76	5.78
19	805.8	211.41	7.9	147.3	150.9	155.2	158.8	442.6	415.3	287.4	256.5	272.0	.2576	5.66	5.67

*Used in computing approximate Transfer Rate

Gas weight=114.7 pounds per hour. Heating surface of one jacket, 0.5236 square feet.

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This formula necessitates a knowledge of the metal temperatures. The method of determining the difference between the metal temperature and the temperature of the circulating water is given in the article on Thermo-couple Tests, page 32, and is expressed by the formula,

$$\Delta t = \frac{H}{49 + 0.253\,W} \qquad (4)$$

The metal temperatures thus determined are used in computing the *average* difference between water and metal temperatures, these average differences being in turn used in formula,

$$R = \frac{H_1}{S(T_m - t_m)} \qquad (7)$$

where S = the surface of each jacket = 0.5238 square feet.

$T_m - t_m$ = the average difference in temperature between metal and circulating water.

The methods of using the approximate rates of heat transfer as determined by this formula have been described. All of the methods of computation of the final heat transfer rates involved in the above formulae are obvious. Table 3, page 58, gives the arrangement of the work for the individual test in question. The specific heats of gas through each jacket as given in this table are the average of the instantaneous specific heats at the gas temperatures entering and leaving the jacket.

Plotted Results

At the completion of the computations the results from each test were plotted, Figs. 10 and 11* being typical of this series of curves. These are drawn with the heat transfer rates as ordinates and the temperature differences as abscissae. Each curve represents one test, and, hence, a constant gas weight.

From these curves a second series of plots was drawn, typically represented by Figs. 12 and 13†. Here the gas weights were changed to rates of gas flow in pounds per hour per square foot of cross-sectional area of the transfer tube, this area being 0.02182 square feet. This series of curves is drawn with the

* Pages 42–43.
† Pages 60–61.

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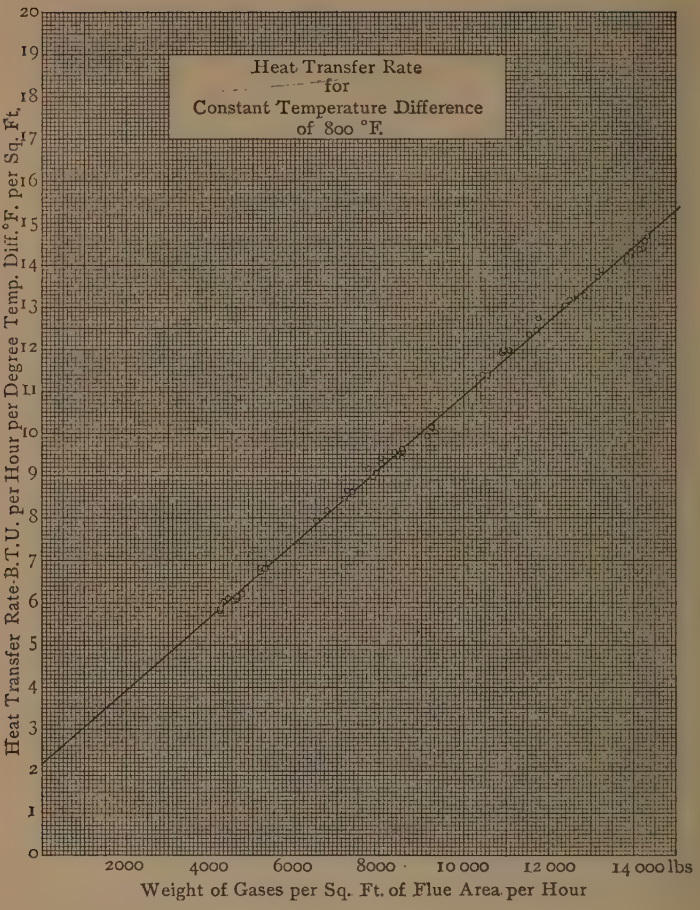

Heat Transfer Rate
for
Constant Temperature Difference
of 800 °F.
Heat Transfer Rate-B.T.U. per Hour per Degree Temp. Diff. °F. per Sq. Ft.
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
2000
4000
6000
8000
10 000
12 000
14 000 lbs
Weight of Gases per Sq. Ft. of Flue Area per Hour

FIG. 12

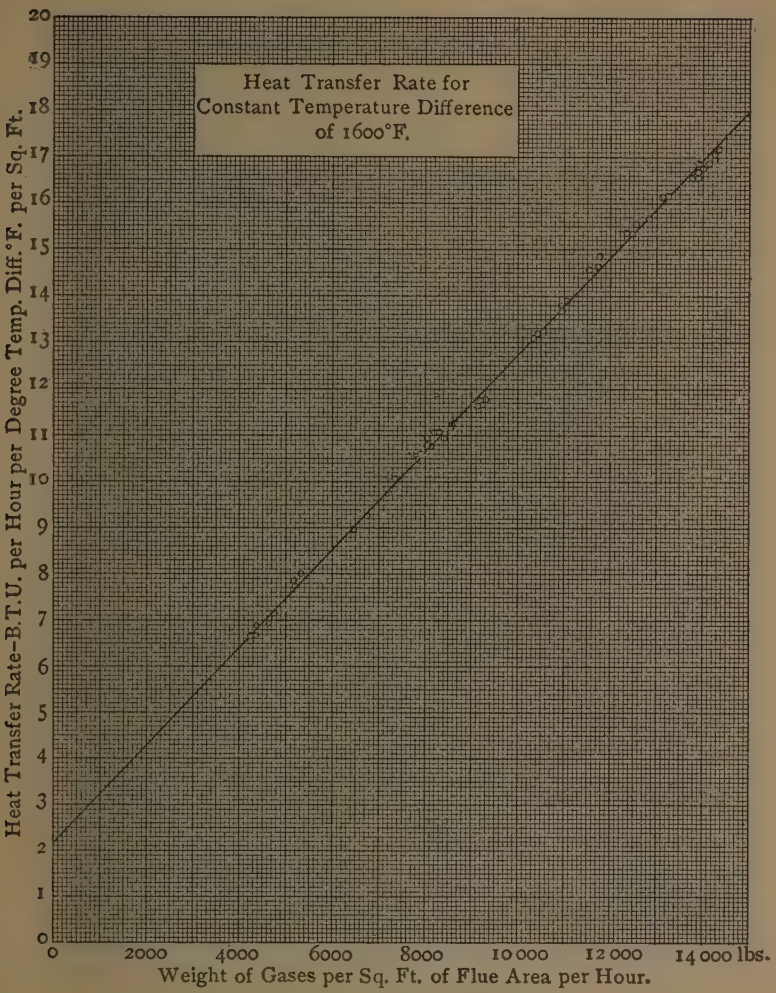

Heat Transfer Rate for
Constant Temperature Difference
of 1600°F.
Heat Transfer Rate–B.T.U. per Hour per Degree Temp. Diff. °F. per Sq. Ft.
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
0
2000
4000
6000
8000
10 000
12 000
14 000 lbs.
Weight of Gases per Sq. Ft. of Flue Area per Hour.

FIG. 13

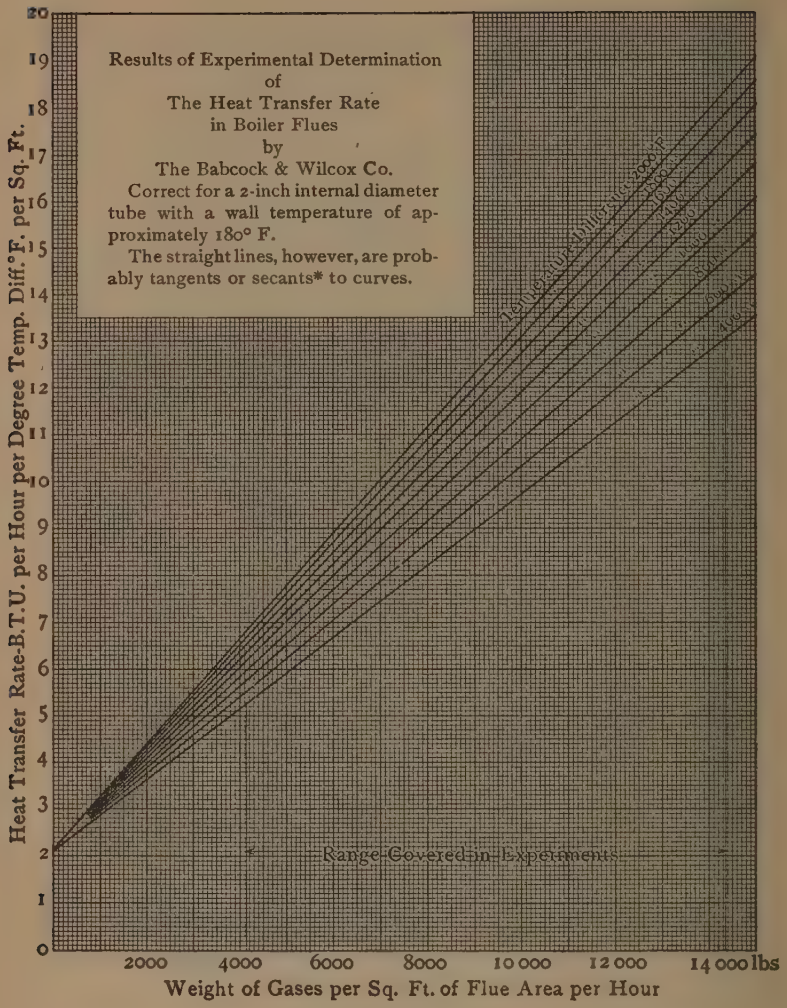


FIG. 14

* See page 66.

heat transfer rates as ordinates and the weights of gas per square foot of cross-sectional area per hour as abscissae. Each of these curves represents a constant temperature difference.

It is to be noted that each curve represents results from all tests due to the fact that every point has been taken from each individual curve for each test previously described. For the sake of accuracy, each of these curves was plotted separately showing all points used in their determination in order to determine the lines independently of each other.

Fig. 14 shows the final results of the present determinations, combining all of the curves represented by Figs. 12 and 13.

SUPPLEMENTARY EXPERIMENTS

AT the conclusion of the experiments and computations described, it was determined to run a series of supplementary experiments to note the effect on the rate of heat transfer of a variation in the linear dimension of the gas passage channel.

As the simplest method of changing this channel without disturbing the existing apparatus, a one-inch outside diameter steel core was inserted in the 2-inch heat transfer tube. This core was held central in the transfer tube by three sets of three ⅛-inch pins screwed into the transfer tube at intervals of 6 feet.

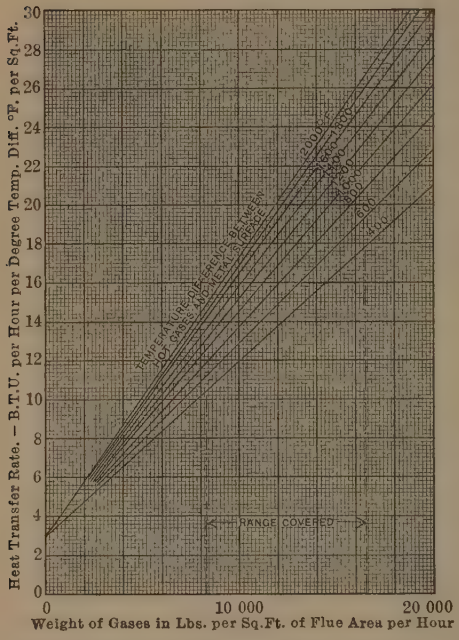


FIG. 15 HEAT TRANSFER RATES FOR A 2-INCH INTERNAL DIAMETER TUBE FITTED WITH 1-INCH EXTERNAL DIAMETER CORE

A series of experiments was then run with the gas passage area reduced by the one-inch core by 0.1636 square feet and gas weights covering a range of from 8300 to 16,500 pounds per square foot of passage area per hour.

This series of experiments was conducted in exactly the same manner as those already described and the computations were made in the same way. The results of this series are shown in graphic form by Fig. 15.

While the tests of the tube equipped with a core were not nearly so numerous as in the primary experiments, the transfer rates shown in Fig. 15 are unquestionably correct for the

particular form of gas passage area used, namely, a 2-inch internal diameter tube fitted with a one-inch outside diameter core. It is not to be assumed, however, that the rates shown in this figure would hold for an uncored tube of a transverse sectional area equivalent to that of the annular 2-inch tube.*

* See Discussion of Results, pages 66–67.

DISCUSSION OF RESULTS

THE final results of the experiments shown in Fig. 14 give the actual heat transfer rates that would exist when the mean temperature of the gases is greater than that of the metal surface by the number of degrees shown on the line corresponding to such temperature difference. These are actual or instantaneous rates of transfer and need no interpretation or correction. The transfer rates have been measured for every point and it is believed they represent the first accurate measurements of the kind made.

The individual plots that have been made of the transfer rates against the rate of gas flow per square foot of passage area for a constant gas temperature difference (as represented by Figs. 12 and 13) indicate that the experiments as a whole are consistent. The points from which this series of curves was plotted were taken from tests run on separate days under unavoidably different conditions. If the results were not uniformly accurate the points would have varied to a great extent and the accuracy is obviously sufficient to give definite values to the constants that must necessarily be used in a heat transfer formula.

It is apparent from these curves of heat transfer and rates of gas flow for a constant temperature difference (as represented by Figs. 12 and 13) that, while to a very close approximation, the equation

$$R = a + b\frac{W}{A}$$

holds, as a matter of fact, the line represented by this equation is a secant or a tangent to a curve which bends from the line in both directions. The range of the experiments with the uncored transfer tube, which were by far the more numerous, was from 4,000 to 14,000 pounds of gas per hour per square foot of gas passage area. Had this range been over a lower set of gas weights, the value of the constant a in this formula would have been smaller and the value of b greater, while had the weight range been higher, the value of a would have been greater and that of b less. It would be possible to take into account this variation from a straight line by the assumption of a third term in the

formula, but for all practical purposes the variation is so small that any such assumption appears unnecessary. The actual curve will, of course, be discontinuous towards the origin, as when the point of critical velocity of the gas in the tube is approached, the motion will change, and the transfer rate diminish as the straight stream line motion sets in.

The supplementary experiments with the one-inch outside diameter core were made, as stated, to determine the effect of a variation in the linear dimension of the gas passage channel.

A core such as was used is heated by the gas in contact with it and cooled by radiation to the surrounding tube. Consequently, it will take a temperature somewhere between the temperature of the gas and that of the tube. Hence, the temperature gradient will be different from that which would have existed had the core been filled with water and the combination used as absorbing surface. The transfer measurements consequently cannot be taken as representing the rates that would exist in a tube having the same mean hydraulic radius but of, say, circular cross section. In a tube of circular cross section, as compared with a larger tube equipped with a core, the transfer rate due to contact would probably be increased, while the actual transfer rate might be decreased on account of radiation from the core.

The supplementary experiments, however, may be safely taken as indicating that over the same range of gas weights per square foot of passage area the constant a in the formula does vary with a variation in the linear dimension of the channel and this, from the dimensions of the quantities involved, is as it should be. To determine definitely the effect of such a change in the linear dimension of the channel on the value of the constant a, it would be necessary to run another series of experiments with a larger or a smaller tube of circular cross section. If such experiments should be conducted in the future, it would be advisable to extend them over a wider range of gas weights. The information desired by The Babcock & Wilcox Company was obtained from the primary series of experiments described in this paper and the suggested future experiments are left to those interested in the further investigation of the laws governing heat transfer.

APPENDIX A

DERIVATION OF CORRECTIVE FORMULA FOR HEAT ABSORPTION

THE equation for the corrected heat absorption in B. t. u. per hour in the individual jackets was derived as follows: The assumed equation for the heat absorbed in any jacket, in terms of the jacket number, for any test, is

$$\log_{10} H_x = a + bx + cx^2 \qquad (6)$$

where H_x = heat absorbed in one jacket in B. t. u. per hour, not corrected for radiation,

x = number of jacket, considering the jacket nearest the furnace No. 1 and counting in the direction of the gas flow,

a, b and c = constants to be determined by methods of least squares.

Since jacket No. 1 was found to be affected by its proximity to the furnace and No. 20 had but one adjacent jacket, the first and twentieth jackets are not included in the following, jacket No. 2 being considered as No. 1 and No. 19 as No. 18.

Multiplying equation (6) by x, we have

$$x \log_{10} x = ax + bx^2 + cx^3 \qquad (18)$$

Multiplying equation (18) by x we have

$$x^2 \log_{10} x = ax^2 + bx^3 + cx^4 \qquad (19)$$

Substituting for x in these three equations all values from 1 to 18, we have three sets of equations. Adding each set separately, these may be written:

$$\Sigma_1^{18} \log_{10} H_x = 18a + b \Sigma_1^{18} x + c \Sigma_1^{18} x^2 \qquad (20)$$

$$\Sigma_1^{18} x \log_{10} H_x = a \Sigma_1^{18} x + b \Sigma_1^{18} x^2 + c \Sigma_1^{18} x^3 \qquad (21)$$

$$\Sigma_1^{18} x^2 \log_{10} H_x = a \Sigma_1^{18} x^2 + b \Sigma_1^{18} x^3 + c \Sigma_1^{18} x^4 \qquad (22)$$

Let

$$\Sigma_1^{18} \log_{10} H_x = L$$

$$\Sigma_1^{18} x \log_{10} H_x = M$$

$$\Sigma_1^{18} x^2 \log_{10} H_x = N$$

But

$$\Sigma_1^{18} x = 171$$

$$\Sigma_1^{18} x^2 = 2109$$

$$\Sigma_1^{18} x^3 = 29241$$

$$\Sigma_1^{18} x^4 = 432345$$

Substituting in (*20, 21* and *22*)

$$L = 18\,a + 171\,b + 2109\,c$$
$$M = 171\,a + 2109\,b + 2924\,c$$
$$N = 2109\,a + 29241\,b + 432345\,c$$

Solving for a, b and c in the last three equations,

$$c = \frac{N - \frac{19}{3}(3M - 10L)}{10336} \qquad (23)$$

$$b = \frac{1}{51}\left(\frac{2}{19}M - L\right) - 19\,c \qquad (24)$$

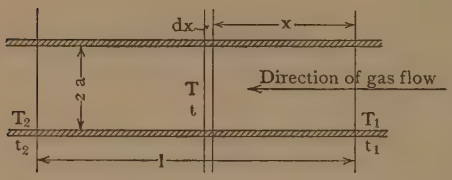
$$a = \frac{1}{3}\left(\frac{M}{57} - 37\,b - 513\,c\right) \qquad (25)$$

These values for a, b and c were then computed for each test and from formula (*6*), the heat absorbed for any jacket x determined.

APPENDIX B

DERIVATION OF HEAT TRANSFER RATE FORMULA

NOTATION



t_1 = Temperature of metal at hot end of jacket (furnace end),

t_2 = Temperature of metal at cold end of jacket,

T_1 = Temperature of gas at hot end of jacket,

T_2 = Temperature of gas at cold end of jacket,

t = Temperature of metal at distance x from hot end of jacket,

T = Temperature of gas at distance x from hot end of jacket,

l = Length of jacket in feet,

a = Radius of jacket in feet,

w = Weight of gas passing through jacket in pounds per hour,

c = Specific heat of gas,

R = Rate of heat tansfer in B. t. u. per hour per degree Fahrenheit difference in temperature per square foot of heating surface.

Since the length of an individual jacket is relatively small as compared with the total length of the transfer tube, the individual jacket may be considered as approaching a differential element of the whole tube. Considering then an individual jacket, the following assumptions are made:

First. The metal temperature throughout any individual jacket varies in accordance with a straight line law. Then

$$t = t_1 - \frac{t_1 - t_2}{l} x \qquad (26)$$

where t = the temperature of the metal at any point x along the jacket.

Second. The specific heat of the gas is constant throughout the length of any individual jacket.

Third. The heat transfer rate throughout any individual jacket is constant.

Since the gases were cooled within the pipe the heat absorbed by the surface must be equal to the heat given up by the gases. Hence the heat absorbed per hour by the surface of a jacket of a length dx must equal the heat given up by the gases in their passage through the distance dx, or

$$R(T-t)\,2\pi a dx = -wc\frac{dT}{dx}dx \qquad (27)$$

Differentiating in equation (26)

$$dt = -\frac{t_1-t_2}{l}dx$$

or

$$dx = -\frac{l}{t_1-t_2}dt$$

Substituting in equation (27)

$$R(T-t)\frac{2\pi al}{t_1-t_2}dt = wcdT$$

Let

$$\frac{R}{wc}\frac{2\pi al}{(t_1-t_2)} = \frac{RS}{wc(t_1-t_2)} = p$$

where $S = 2\pi al$, the heating surface of a jacket in square feet.

Then

$$(T-t)p = \frac{dT}{dt}$$

or

$$\frac{dT}{p} - Tdt = -tdt$$

Integrating

$$T = t + \frac{1}{p} + Ce^{pt}$$

or

$$\log_e\left(T - t - \frac{1}{p}\right) = \log_e C + pt$$

When

$$T = T_1,\ t = t_1$$

and

$$T = T_2,\ t = t_2$$

Hence

$$\log_e\left(T_1 - t_1 - \frac{1}{p}\right) = \log_e C + pt_1$$

and

$$\log_e\left(T_2 - t_2 - \frac{1}{p}\right) = \log_e C + pt_2$$

Subtracting $$\log_e\left(\frac{T_1-t_1-\frac{1}{p}}{T_2-t_2-\frac{1}{p}}\right)=p(t_1-t_2)$$

But $$p=\frac{RS}{wc(t_1-t_2)}$$

Substituting $$\log_e\left(\frac{T_1-t_1-(t_1-t_2)\frac{wc}{RS}}{T_2-t_2-(t_1-t_2)\frac{wc}{RS}}\right)=\frac{RS}{wc}$$

or $$R=\frac{wc}{S}\log_e\left(\frac{T_1-t_1-(t_1-t_2)\frac{wc}{RS}}{T_2-t_2-(t_1-t_2)\frac{wc}{RS}}\right)$$

which is the equation used in computing the heat transfer rates of individual jackets.

APPENDIX C
SPECIFIC HEAT OF GASES

SINCE the weight of the gases and their mean temperatures at various points throughout the transfer tube were determined from the specific heats of the various constituents, the importance of a proper knowledge of their correct values is obvious.

Holborn and Henning determined, by the method of constant pressure calorimetry, the specific heat of carbon dioxide, nitrogen and water vapor up to a temperature of 1450 degrees centigrade (2642 degrees Fahrenheit). Their values for carbon dioxide and nitrogen, over the temperature range of their experiments, are unquestionably authoritative and have been accepted for these tests. For water vapor, values have been derived from Marks and Davis' Steam Tables, which are in part based on the experiments of Holborn and Henning. These values also are unquestionably very nearly correct. For the specific heat of oxygen a formula has been derived which is in close agreement with existing data.

The general formula for the specific heat of a gas at constant pressure is given by the function

$$c_p = A + Bt + Ct^2 \qquad (28)$$

The mean specific heat of a gas between the temperatures t_1 and t_2 will then be

$$c_p = \int_{t_1}^{t_2} \frac{(A + Bt + Ct^2)}{(t_2 - t_1)} dt$$

or by integration

$$c_p = A + \tfrac{1}{2}B(t_2 + t_1) + \tfrac{1}{3}C[(t_2 + t_1)^2 - t_2 t_1]$$

The specific heat formulae given by Holborn and Henning are expressed in terms of centigrade scale, while the values used in the present experiments are required in terms of the Fahrenheit scale. The following methods of conversion have been used:

If $c_p = a + b\theta + c\theta^2$, where θ is the temperature referred to the centigrade scale, and $A + Bt + Ct^2$, the corresponding values referred to the Fahrenheit scale, we have, since $\theta = \frac{5}{9}(t - 32)$,

$$c_p = A + Bt + Ct^2 = a - \frac{5}{9}32b + \frac{25}{81}32^2 c + (\frac{5}{9}b - 2\frac{25}{81}32c)t + \frac{25}{81}ct^2$$

Hence, the values of the constants in the specific heat formulae referred to the Fahrenheit scale are

$$A = a - 32\frac{5}{9}b + \frac{25}{81}32^2 c$$

$$B = \frac{5}{9}b - 2\frac{25}{81}32c$$

$$C = \frac{25}{81}c$$

For formulae in which there is no third term (for nitrogen and oxygen), *i. e.*, where $c=o$

$$A = a - 32\frac{5}{9}b$$

$$B = \frac{5}{9}b$$

Carbon Dioxide—Holborn and Henning in the "Annalen der Physic," 1907, No. 10, give the value of the mean specific heat of carbon dioxide at constant pressure as

$$c_{p_{o-\theta}} = 0.2010 + 0.0000742\theta - 0.00000001 8\theta^2$$

and the value of the instantaneous specific heat as

$$c_{p_\theta} = 0.2010 + 0.0001484\theta - 0.000000054\theta^2$$

This formula holds good up to 1450 degrees centigrade.

Expressed in terms of the Fahrenheit scale in accordance with the relation just given the instantaneous specific heat at constant pressure is

$$c_{p_t} = 0.1983 + 835 \times 10^{-7} t - 16.7 \times 10^{-9} t^2 \qquad (11)$$

The values of the specific heat as determined by Holborn and Henning's converted formula (*11*), when plotted were found to fall off rapidly for values of t above 2400 degrees Fahrenheit. The work on the determination of authentic carbon dioxide specific heat values was done previous to the running of the actual experiments. It was believed at this time that temperatures up to 3000 degrees Fahrenheit would be encountered in the experiments and for this reason it appeared necessary to introduce new data that would give values as nearly as possible correct for temperatures above 2400 degrees Fahrenheit. As a matter of fact, inasmuch as the jacket nearest the furnace was later disregarded

in making computations, but few temperatures above 2500 degrees Fahrenheit were recorded and the formula derived for higher temperatures was but rarely used. However, as the question is of interest in considering the general subject of the Specific Heat of Gases, it is included here.

Mathias Pier gives in the "Electro-Chemical Zeitschrift," Nov. 1910, Vol. 16, an account of his experiments with carbon dioxide at high temperatures. He proposes a formula based on his own observations in conjunction with the experiments of Holborn and Henning which gives as the mean molecular specific heat at constant volume

$$mc_{v_{0-\theta}} = 6.80 + 3.3 \times 10^{-3}\theta - 0.95 \times 10^{-6}\theta^2 + 0.1 \times 10^{-9}\theta^3$$

Pier's observations were over a temperature range of 1611 degrees to 2110 degrees centigrade. At two temperatures, 1831 degrees and 2110 degrees centigrade the values of mc_v, are respectively 10.27 and 10.47.

If m=the molecular weight of the gas

$$m(c_p - c_v) = 2.0 \qquad (29)$$

Holborn and Henning's formula, expressed in terms of mean molecular specific heat at constant volume, referred to the centigrade scale, becomes

$$mc_{v_{0-\theta}} = 6.844 + 0.003265\theta - 0.00000792\theta^2$$

when θ=1000 deg. C., mc_v=9.137

θ=1350 deg. C., mc_v=9.808

From these values of mc_v at 1000 degrees and 1350 degrees centigrade, and those of Pier at 1831 degrees and 2110 degrees centigrade, the values of the constants in the formula $mc_v = a + b\theta + c\theta^2 + d\theta^3$ were determined, which gave as a formula for the mean molecular specific heat at constant volume, at high temperatures

$$mc_{v_{0-\theta}} = 6.877 + 3.4 \times 10^{-3}\theta - 1.1 \times 10^{-6}\theta^2 + 0.14 \times 10^{-9}\theta^3$$

or from (29), at constant pressure

$$mc_{p_{0-\theta}} = 8.877 + 3.4 \times 10^{-3}\theta - 1.1 \times 10^{-6}\theta^2 + 0.14 \times 10^{-9}\theta^3$$

Specific Heat of
Carbon Dioxide
Holborn and Hennings values to 2200° F.
Above 2200° values from formula (30)
— — Shows Holborn and Hennings values above 2200° F.
Instantaneous Specific Heat at Constant Pressure
.30
.29
.28
.27
.26
.25
.24
.23
.22
.21
.20
.19
200
400
600
800
1000
1200
1400
1600
1800
2000
2200
2400
2600
2800
3000
Temperature Degrees Fahrenheit

Expressed in terms of mean specific heat this becomes

$$c_{p_{o-\theta}} = .2018 + 772 \times 10^{-7}\theta - 25 \times 10^{-9}\theta^2 + 3.2 \times 10^{-11}\theta^3$$

and in terms of instantaneous specific heat

$$c_{p_\theta} = .2018 + 1544 \times 10^{-7}\theta - 75 \times 10^{-9}\theta^2 + 12.8 \times 10^{-11}\theta^3$$

Converted to terms of the Fahrenheit scale, in which form it is used in these experiments, we have

$$c_{p_t} = .1991 + 873 \times 10^{-7}t - 23.4 \times 10^{-9}t^2 + 12.8 \times 10^{-11}t^3 \qquad (30)$$

A comparison of values for temperatures above 2200 degrees Fahrenheit, as determined by this formula, by Holborn and Henning's formula, and by Pier's formula, is of interest:

Temperature Degrees Fahrenheit	*By Formula (30)*	*Holborn and Henning*	*Pier*
2200	.3013	.3013	.3023
2400	.3042	.3027	.3055
2600	.3066	.3027	.3081
2800	.3083	.3014	.3101
3000	.3098	.2988	.3114

Fig. 16 gives in graphic form the instantaneous specific heat of carbon dioxide at various temperatures. In plotting this curve Holborn and Henning's values were used up to 2200 degrees Fahrenheit, and above this temperature values determined from formula (*30*).

NITROGEN—Holborn and Henning in the paper to which reference has been made give for the mean specific heat of nitrogen at constant pressure, the value

$$c_{p_{o-\theta}} = 0.2350 + 0.000019\theta$$

and the instantaneous value

$$c_{p_\theta} = 0.2350 + 0.000038\theta$$

Converted to terms of the Fahrenheit scale this becomes

$$c_{p_t} = 0.2343 + 0.000021t \qquad (13)$$

Holborn and Henning's observations extended to a temperature of 1347 degrees centigrade or 2456 degrees Fahrenheit.

Since there were but few calculations in the present experiments involving temperatures above this, it was not considered necessary to introduce any additional data.

Oxygen—The existing data on the specific heat of oxygen is meagre.

Holborn and Austin experimented with oxygen mixed with 9 per cent nitrogen. They give for the mean specific heat of pure oxygen the following values:

10 degrees to 200 degrees centigrade, 0.2175 (Regnault's value)
20 degrees to 440 degrees centigrade, 0.2240
20 degrees to 630 degrees centigrade, 0.2300

These authorities state that if the value 0.2375 for air is to be taken as correct, it would appear from their experiments between 25 degrees and 250 degrees centigrade that Regnault's value 0.2175 should be increased to 0.2206.

Langen and Pier investigated oxygen at high temperatures, and give for the mean molecular specific heat at constant volume

$$mc_{v_{0-\theta}}=4.80+0.0006\theta \text{ (Langen)}$$

$$mc_{v_{0-\theta}}=4.90+0.00045\theta \text{ (Pier)}$$

The formula proposed for the mean specific heat of oxygen at constant pressure is

$$c_{p_{0-\theta}}=0.216+0.000017\theta \qquad (31)$$

and for the instantaneous specific heat

$$c_{p_{\theta}}=0.216+0.000034\theta \qquad (32)$$

The mean specific heats calculated by formula (31) as compared with Holborn and Austin's values are

Degrees Centigrade	*Formula (31)*	*Holborn and Austin*
10 to 200	0.2196	0.2175
20 to 440	0.2238	0.2240
20 to 630	0.2271	0.2300

The proposed formula expressed in terms of mean molecular specific heat is

$$mc_{v_{0-\theta}}=4.91+0.00054\theta$$

as compared with Langen and Pier's values, $4.80+0.0006\theta$ and $4.90+0.00045\theta$ respectively.

From these comparisons the proposed formula apparently gives higher values for high temperatures than do Langen and Pier, but the proposed values agree more nearly with those of Holborn and Austin, which are practically the only values given for temperatures below 1000 degrees centigrade.

The proposed formula for the instantaneous specific heat of oxygen at constant pressure, expressed in terms of the Fahrenheit scale, which has been used in these experiments is then

$$c_{p_t}=0.2154+0.000019t \qquad (12)$$

Water Vapor—The formula used for the instantaneous specific heat of water vapor is

$$c_{p_t}=0.4541+32 \times 10^{-7}t+2825 \times 10^{-11}t^2 \qquad (14)$$

This formula is based on values given in Marks and Davis' Steam Tables and is derived from their values of the total heat of superheated steam* as follows:

The partial pressure of the water vapor in the flue gases was approximately one pound absolute.

If the instantaneous specific heat is expressed by the function $A+Bt+Ct^2$, the heat absorbed between the temperatures t_1 and t_2 will be

$$H_2-H_1=\int_{t_1}^{t_2}(A+Bt+Ct^2)dt$$

or, integrating

$$H_2-H_1=A(t_2-t_1)+B\left(\frac{t_2^2-t_1^2}{2}\right)+C\left(\frac{t_2^3-t_1^3}{3}\right)$$

For one pound absolute

$H_2=1104.4$ B. t. u. $\quad t_1=101.7$ deg. F.

Then

$$H_2-1104.4=A(t_2-101.7)+B\left(\frac{t_2^2-101.7^2}{2}\right)+C\left(\frac{t_2^3-101.7^3}{3}\right) \qquad (33)$$

When

$t_2=101.7$ deg. F. $\quad H_2=1195.6$ B. t. u.
$t_2=1101.7$ deg. F. $\quad H_2=1573.0$ B. t. u.
$t_2=2101.7$ deg. F. $\quad H_2=2107.0$ B. t. u.

* See pages 22, 23, and 66 of Marks and Davis' Steam Tables.

Substituting these values in (*33*) and solving, we have

$$A = 0.4541$$
$$B = 32 \times 10^{-7}$$
$$C = 2825 \times 10^{-11}$$

or for the instantaneous specific heat of water vapor at a constant pressure of one pound absolute, formula (*14*).

APPENDIX D

CALIBRATION OF THERMO-COUPLES

THE thermo-couples described were calibrated in the laboratory of The Babcock & Wilcox Co., the apparatus used for this purpose being shown in Fig. 17.

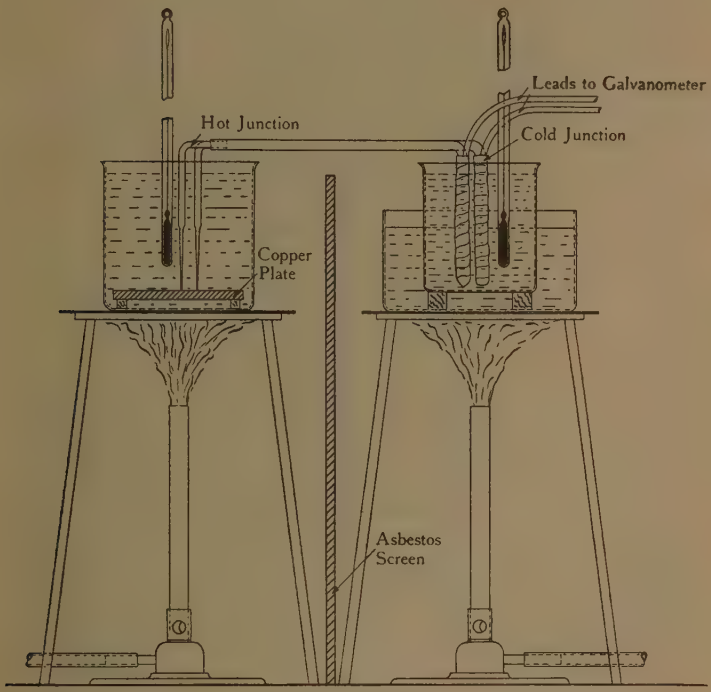


FIG. 17. THERMO-COUPLES ARRANGED FOR CALIBRATION

The hot ends of the couple were firmly held against a polished copper slab placed in a glass breaker. This copper slab was raised from the bottom of the breaker by small blocks of wood, in order that the metal might be maintained at a constant temperature throughout. The cold ends were placed in a double water bath as shown, to secure a constant cold end temperature. Thermometers

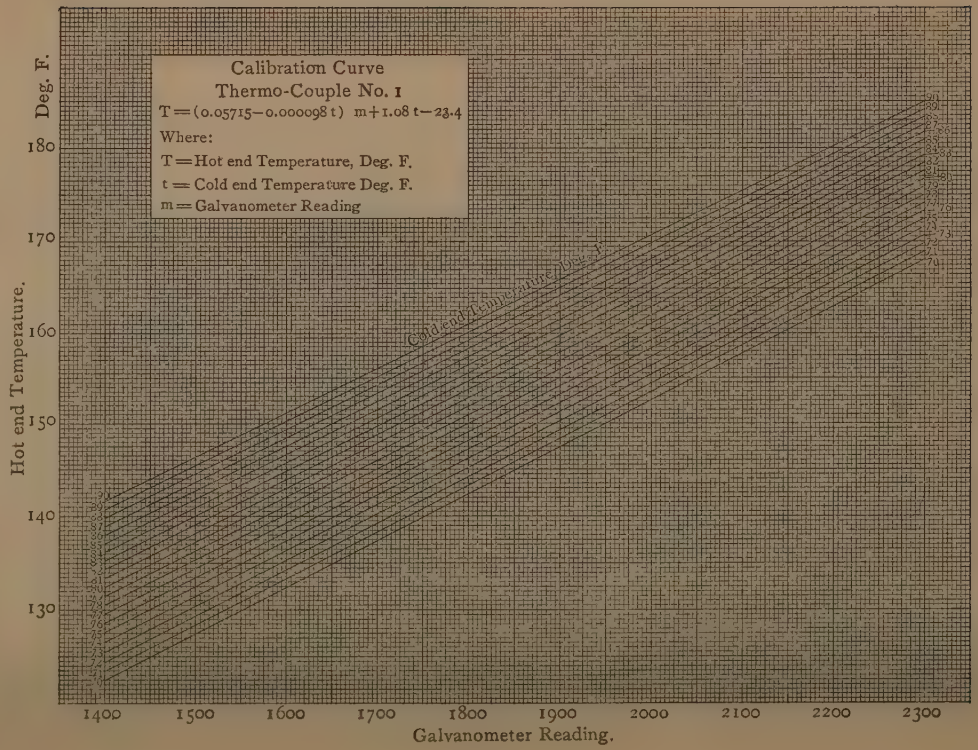

Calibration Curve
Thermo-Couple No. 1
T = (0.05715 − 0.000098 t) m + 1.08 t − 23.4
Where:
T = Hot end Temperature, Deg. F.
t = Cold end Temperature Deg. F.
m = Galvanometer Reading
Hot end Temperature, Deg. F.
180
170
160
150
140
130
Cold end Temperature Deg. F.
1400
1500
1600
1700
1800
1900
2000
2100
2200
2300
Galvanometer Reading.

FIG. 18

were placed with their bulbs as close as possible to the hot junction and the copper slab, and to the cold ends, in order to avoid any error due to circulation of water in the containing vessels. Bunsen burners were used to heat the two receptacles and an asbestos screen between the two prevented errors due to convection currents.

The cold junction water bath was first heated, a steady temperature being maintained by the use of a flame which gave just sufficient heat to balance the radiation losses. Heat was then applied to the hot end receptacle until the lowest temperature of the calibration was reached. The flame was then removed and a reading taken as soon as the temperature started to lower, a simultaneous reading being taken of the cold end thermometer and of the galvanometer. The flame was again applied until a higher temperature was reached, and again removed and another set of readings taken. The hot end readings were observed on the falling temperature to obviate any error from a possible lagging action of the galvanometer. In this manner the whole range of hot end temperatures was covered, while the cold end temperature was maintained constant. The same operation was then repeated for the different cold end temperatures covering the required range.

From the data thus obtained four series of calibration curves were plotted for the two ranges of cold end temperatures of each couple, having as ordinates the hot end temperatures, and as abscissae the galvanometer readings. Fig. 18 shows a typical set of these calibration curves. From these plots the following equations were computed, in which

T=hot end temperature in degrees Fahrenheit
t=cold end temperature in degrees Fahrenheit
m=galvanometer reading

Couple No. 1

Contact range

T=130 deg. F. to T=150 deg. F.
t= 70 deg. F. to t= 90 deg. F.

$$T=(0.05715-0.000098t)m+1.08t-23.4$$

Tube temperature range

T=140 deg. F. to T=212 deg. F.
t=130 deg. F. to t=150 deg. F.

$$T=0.00436m+0.8583t+6.4$$

Couple No. 2

Contact range

$T+130$ deg. F. to $T=150$ deg. F.

$t=70$ deg. F. to $t=90$ deg. F.

$$T=(0.0598-0.00014t)m+1.195t-32.7$$

Tube temperature range

$T=140$ deg. F. to $T=212$ deg. F.

$t=130$ deg. F. to $t=150$ deg. F.

$$T=0.0417m+0.960t-7.4$$

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